A COGNITIVE LOAD-CENTRIC INQUIRY INTO MATHEMATICAL PROBLEM SOLVING IN MODERATE- TO HIGH-ABILITY STAGE 4 STUDENTS: AN EXPLORATORY STUDY

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Statement of Originality

This is to certify that all work contained in this thesis is my own unless otherwise cited. This thesis has not been previously submitted in whole or part requirements towards a degree at this or any other University.

Signed  Date 26/11/09

Acknowledgements

Sweet, this is the only part of the thesis in which I can actually type as myself! I'll try to keep it to a page.

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Abstract

This research project examines problem solving processes in moderate to high-ability grade 7 and 8 mathematics students using cognitive load theory as a theoretical basis. This was done using a qualitative method and thematic analysis to identify the theme of cognitive load as it arose during four audio-recorded interview sessions with the six participants. A model is presented that examines the problem solving process based on the data collected in this study. This model proposes that problem solving involves reading, internalisation and classification of a problem. The solver uses mental restructuring and solution strategies to reduce cognitive load. Metacognition is used as a cognitive load monitoring device. Prior knowledge largely determines the chosen solution path and strategies. Unresolved cognitive overload results in confusion which leads to non-solution; cognitive clarity results in a solution. The implications of this model are discussed for teachers and students.

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Chapter 1

Introduction

1.1 Background

The human mind is capable of incredible plasticity and flexibility. The ability to think, to feel, to solve problems and to learn is one of life's great wonders. Since the late 19th century, from modern psychology's beginning through such early pioneers such as James and Ebbinghaus, memory and the human cognitive architecture have been systematically decompiled and modelled, all the while developing a deeper understanding of how the human mind works. Miller (1956) and de Groot's (1966, as cited by Sweller, 1999) landmark papers on the role of working memory and long-term memory, on individuals' abilities to develop expertise and solve problems have given rise to deep investigations in the area of cognitive psychology. In the early 1980's, researchers such as John Sweller were concerned with the thought of why some material might be easy to learn and other material might be difficult. Through many years of research and refinement, cognitive load theory was born. Cognitive load theory has been instrumental in giving makers of instructional material a theoretical basis upon which to design their aids for learning. However, cognitive load theory should not necessarily be locked into this single area of applicability. Cognitive load theory also has significant explanatory power in the area of problem solving.

This study uses cognitive load theory as the basis for an analysis into the problem solving processes of students of high mathematical ability. Through detailed observation of six lower high school students, audio and problem solving data was collected and analysed. The study sought to answer the following four research questions:

1. Can cognitive load theory be used to explain observations of student problem solving performance?

2. If so, can a model incorporating cognitive load theory be created to guide teaching for problem solving?

3. What implications does the model hold for teachers?

4. What implications does the model hold for students?

1.2 Significance of the Study

Although there is a significant body of literature relating to problem solving, models of the human cognitive architecture, and cognitive load theory, this literature is overwhelmingly quantitative. This study aims to use qualitative data from high-ability mathematics students to explore in-depth the participants' problem solving experiences. The study proposes a model of problem solving which can be used to inform teachers of the process of solving mathematical problems among students at the early high school level.

Cognitive load theory is a powerful theory that was born through research on problem solving and the human cognitive architecture. However, the combination of the use of qualitative methods and cognitive load theory to re-examine solution paths chosen by relatively able young lower high school students has the potential to produce new insights.

1.3 Structure of the Study

This is a study uses four audio-recorded sessions; a Raven's Matrices test, two problem solving, and an interview, to investigate the effect of cognitive load on the solution process. These data collection methods were selected to ensure a comprehensive coverage of issues surrounding problem solving from a number of perspectives. The data was collected from six students at two private schools and then analysed and reported according to qualitative methods. Finally, the data was used to produce a model of problem solving that has implications for teachers and students.

1.4 Structure of the Thesis

Chapter two examines the literature in relation to models of memory and the human cognitive architecture, cognitive load theory and problem solving in gifted, high-ability, expert and novice students.

Chapter three outlines the method of the study, introduces the participants, restates the research questions and describes the means of data collection and analysis. It also describes the procedures for gaining ethical clearance and permission to conduct the study.

Chapter four presents the results in detail and argues for a model describing the problem solving process.

Chapter five examines the potential implications for teachers, summarises the results of the study, outlines the study's limitations and recommends potential areas for further research.

1.5 Definition of Terms

Cognitive load	The total load imposed on working memory at any one time.
Element interactivity	The level of interrelatedness of a problem's disparate elements of information.
Expertise	Expertise involves superior performance based upon extensive knowledge built through contact with the domain of expertise over a long period of time.
Goal state	The final state of the problem in which a correct solution has been reached.
Intrinsic cognitive load	The load imposed on working memory that is intrinsic to the problem due to its element interactivity.
Long-term memory	The component of human memory that is of virtually unlimited capacity and duration.
Metacognition	A person's knowledge concerning their own cognitive processes or anything related to those processes.
Problem state	The current configuration of problem elements – the state of solution at which the problem is.
Scaffolding	The act of providing support to promote learning and performance, by giving timely and precise feedback.

Schema	"A cognitive construct that permits us to treat multiple elements of information as a single element" (Sweller, 1999, p. 10).
Self-efficacy	A person's belief in their ability to complete a task with a successful outcome.
Solution path	The sequence of steps one follows to arrive at a solution for a problem. Problems may have more than one correct solution path.
Working memory	The component of human memory that deals with conscious thought. While it has a very limited capacity and duration, it is also very flexible.

Chapter 2

Literature Review

2.1 Introduction

The development of cognitive load theory could be said to commence with Miller's (1956) landmark paper on human capacities for processing information. Miller (1956) stated that, on average, people are limited to being able to hold around only seven *chunks* of information for only a short time in short-term memory. He found that it was not the chunk size, however, that influenced the amount of information that could be stored in short-term memory, but rather the number of chunks (Miller, 1956; Cowan, 1988). Therefore, the larger the chunk size, the more information is present in working memory (the subtle distinction between short-term and working memory will be discussed later in this chapter). For example, by rearranging these seven bits, *e t e l m n e*, into the word *element*, people are able to hold the seven individual bits as one chunk in working memory. Now, what constitutes a chunk for one person may not constitute a chunk (a single word) for another person (Bruning, Schraw, Norby & Ronning, 2004). The ability to chunk information gives rise to schemas in one's long-term memory, which is of virtually unlimited duration and capacity (Bruning et al., 2004). A schema is a composite unit of information whose conceptual elements are strongly related to each other (Anderson, 2000).

2.2 Human Cognitive Architecture

The human cognitive architecture is a topic that has concerned academics for centuries, in the areas of philosophy, psychology, education, psychiatry and biological science (Squire & Kandel,

1999). While there are many models of the human cognitive architecture, there are two closely related ones that I would like to describe. These models have anatomical representations, but for the purposes of this study, I would like to restrict myself to exploring only the functional aspects of the models.

2.2.1 Declarative and Non-Declarative Memory

The first is the model of declarative and non-declarative memory (Squire & Kandel, 1999; Squire & Zola, November 1996, p. 13516). This has been proposed by a number of thinkers (such as Ebbinghaus, Matthew, McDougall and Ryle) in slightly different forms and under different terms (such as implicit/explicit memory; knowing what/knowing how) (see Squire & Kandel, 1999). Declarative memory deals with knowledge of events and facts that can be declared to another, while nondeclarative memory deals with the ability to automatically perform skills. Declarative memory is conscious, while nondeclarative memory is nonconscious (Cowan, 1988, p. 172; Squire & Kandel, 1999). According to Wolfe and Nevills (2004, p. 75), "the difference between declarative and nondeclarative memory is that when skills are mastered in nondeclarative memory, they are automatically and unconsciously performed, reducing the cognitive load." The speed and accuracy advantages of the use of automatic processes that are present in nondeclarative long-term memory over deliberate processes that must occur in declarative or working memory are well publicised (Leahy & Sweller, 2005; Kester, Kirschner & van Merriënboer, 2004; Sweller, 2005; Sweller, van Merriënboer & Paas, 1998). However, the conscious control of processing in declarative memory permits flexibility and creativity (Squire & Kandel, 1999).

2.2.2 Alternative Conceptions of Memory Systems

While Squire and Kandel's model distinguishes memory about facts from memory about automated processes, there are many models of conscious declarative memory that distinguish between a short duration working memory and long-term memory. One of these models is described by Baddeley (2001). While this model will be discussed in more detail in section 2.4, for this section it suffices to say that this model suggests an interchange of information between working and long-term memory, both of which are "separate systems, although they are closely integrated in operation." (Baddeley, 2004, p. 18). This is not the only conceptualisation of the interchange of information between working memory and long-term memory. There are other models of memory, such as that proposed by Cowan (1988), which considers working memory to include an activated subset of long-term memory. As Cowan (1988) states, "a sensible alternative to the original conception of stores is one in which short-term storage consists of the elements within the long-term store that are currently in a heightened state of activation." Thus, in the Cowan (1988) model, there is no need to transfer a duplicate copy of long-term memory schemas into working memory. Working memory directly uses schemas in long-term memory. The process by which schemas are activated for use in working memory, selective attention, performs an equivalent role to the central executive in the Baddeley (2001) model. The limitations of working memory capacity in the Cowan (1988) model arise from a limited capacity to direct and maintain attention. In this view, attention not only involves the maintenance of focus involving sensory input, but also the continuance of activation of select elements of long-term memory (Engle & Oransky, 1999; Squire & Kandel, 1999).

Functionally, working memory may appear as a single entity, but physically, it has been found that elements of memory are distributed throughout the brain rather than residing in a central

area of memory function. This conclusion was largely reached through the work of Karl Lashey and Donald Hebb. Lashey was unable to find a centralised store for memory and concluded that many parts of the brain must be involved in memory retention (Squire & Kandel, 1999). Hebb later suggested that neurons and their vast networks of connections were responsible for the representation of information. A widely agreed notion is that while none of the individual sections of the brain is responsible for memory as a whole, each part has an important and largely unique part to play. Thus, working memory is a distributed functional entity rather than a strictly physical structure (Squire & Kandel, 1999). Now, while these distinctions are notable, the most important feature of working memory in the context of the proposed study is its limited capacity to activate many items simultaneously.

2.3 Schema Theory

Schema theory, which treats chunks of information in long-term memory as complex interrelated representations of concepts, has greatly informed our understanding of learning processes (Bruning et al., 2004). It is this powerful theory that allows the explanation of many phenomena that are observed by memory researchers. In schema theory, learning is seen as "the acquisition of increasingly automated schemas held in long-term memory" (Sweller, 1999, p. 36). Schemas vary in their level of complexity, with experts having more sophisticated and comprehensive schemas in their area of expertise than novices (van Merriënboer & Sweller, 2005). The phenomenon that is most relevant to this study is experts' ability to solve problems more efficiently than novices due to experts' vast networks of rich schemas in long-term memory (Bruning et al., 2004; Sweller, 1988).

Research by de Groot (1966, as cited by Sweller, 1999), and Chase and Simon (1973) suggests that the superior performance of chess grandmasters comes from an automatic ability to recall multiple board configurations, arising from the position of chess pieces on a chessboard and their associated movement abilities, based on past experience. Each configuration has a limited number of quality moves associated with it. Each configuration and its related responses can be regarded as a "chunk" of information – a schema. It is the acquisition of multiple configurations and their related responses, internalised by repeated exposure to real-game situations, which create the chess expert. These experts have developed automated schemas that allow them to achieve oversight of a problem space (in this case, a board configuration). The finding that expertise comes from extensive and rich schemas has been consistently replicated in the areas of electronics, baseballers and high school mathematics students. However, this expertise is highly task-specific. In all cases, it was found that the experts were able to readily recall meaningful problem spaces (e.g. in-game board configurations), but had no better recall than novices on random problem spaces (Larkin, McDermott, Simon & Simon, 1980; Sweller, 1999; Sweller, van Merriënboer & Paas, 1998). This demonstrates that the experts' advantage was caused by prior knowledge. As the scarcity of working memory resources is a major concern in learning and problem solving (Larkin et al., 1980), the importance of building rich, automated schemas in long-term memory is paramount in ensuring students are able to solve problems efficiently.

2.4 Working Memory

Short-term memory has been largely superseded as a construct by working memory, which is essentially short-term memory with an added central executive that controls the attention, processing and data flow of the activated elements of short-term memory (Conway, Cowan, Bunting, Therriault & Minkoff, 2001; Engle, Tuholski, Laughlin & Conway, 1999). Baddeley

(2001) has built on the groundwork of schema theory and Miller's (1956) work on short-term memory through his own model of working memory. Baddeley (2001) proposed a model of working memory based around an information-processing paradigm. His model now contains four separate but interrelated sub-systems (Baddeley, 2001; Bruning et al., 2004). Baddeley suggests that working memory contains a phonological loop, episodic buffer and visual-spatial sketchpad as slave systems, along with an executive control system that coordinates the functioning of these systems (Baddeley, 2001). As previously stated, it is this executive control system that principally differentiates working memory from short-term memory (Engle et al., 1999). The construct of working memory is continuously being refined and developed and as such, is a deep and rich area of on-going research.

The construct of working memory is of central importance to the topic of this thesis, and so should be examined in more detail. Gazzaley, Sheridan, Cooney and D'Esposito (2007, p. 532) define working memory as "the temporary representation of information that was just experienced... but is no longer accessible in the external environment". However, working memory plays a deeper role in cognition than a mere information storage area would. Working memory capacity, as measured by complex span working memory tasks, has been shown to strongly predict general fluid intelligence [1] (Engle et al., 1999; Kane et al., 2004; Schmiedek, Hildebrandt, Lövdén, Wilhelm & Linderberger, 2009). There is also a strong relationship evident between working memory capacity tests and non-verbal tests of general fluid intelligence such as the widely accepted Ravens Advanced Progressive Matrices (RAPM)

[1] The distinction between fluid intelligence and crystallized intelligence roughly equates to reasoning [FI] versus knowledge [CI] (Beauducel & Kersting, 2002). In this study, selecting students of high problem solving ability as measured by the Ravens test represents high fluid intelligence, while selecting students of high test performance represents high crystallized intelligence. The researcher acknowledges that both kinds of intelligence will have an effect on the problem solving styles and approaches used by the students.

(Conway et al., 2002; Kane et al., 2004; Neisser et al., 1996). The RAPM test is particularly relevant to this study as it measures the ability to inductively identify abstract relationships and to simultaneously manipulate two dimensions of pictorial and relational information in memory. The RAPM test score correlates highly with general fluid intelligence (Carpenter, Just & Shell, 1990).

Working memory capacity is scarce (Larkin et al., 1980; van Merriënboer & Sweller, 2005), and can be thought of as the 'bottleneck' of a person's cognitive system (Schnotz & Kürschner, 2007). The scarcity of working memory resources has a strong impact on a person's ability to solve problems. Further, factors as diverse as distractions (Beaman, 2004; Gazzaley et al., 2007; Lecerf & Roulin, 2009), lack of sleep (Gradisar, Terrill, Johnston & Douglas, 2008), handwriting speed (Peverly, 2006) and even food cravings (Kemps, Tiggemann & Grigg, 2008) can have a detrimental effect on available working memory capacity. The situation is such that if the distractors are strong enough, cognitive overload occurs and task failure soon follows.

2.5 Cognitive Load Theory

Miller (1956) and Baddeley's (2001) research on working memory has been combined with the literature on problem solving to form a new area of inquiry into the human cognitive architecture known as cognitive load theory (see Bruning et al., 2004; Chandler & Sweller, 1991; Sweller, 1999). Cognitive load theory is firmly centred on a conceptualisation of cognition that considers working memory resources to be limited and seeks to use strategies that optimise the use of these resources in the processes of learning and problem solving (van Merriënboer & Sweller, 2005). According to Chandler and Sweller (1991, p. 2), "cognitive load theory is concerned with the manner in which cognitive resources are focused and used during

learning and problem solving." Cognitive load theory is generally applied to the area of instructional design. It seeks to create instructional materials that maximise the amount of cognitive load used to acquire and automate schemas, while minimising the cognitive load used in activities that are detrimental to schema acquisition and automation. Any cognitive capacities that need to be directed towards helping the learner make sense of the instructional material are cognitive capacities that are consequently unavailable for schema acquisition and automation (Sweller, 1999).

2.5.1 Forms of Cognitive Load - Intrinsic

The task complexity of a problem depends on the level of interactivity of the problem elements. This assertion has been made both in the research of cognitive load theorists (Sweller, 1994; Sweller, 1999) and other researchers (Schweizer, 2001). This task complexity is intrinsic to the expertise of the learner and the problem itself, and was originally considered unalterable (Sweller, 1994; Sweller, 1999; van Merriënboer & Sweller, 2005). However, it is now believed that instructional design can manipulate this form of cognitive load (Moreno, 2006), known as *intrinsic* cognitive load. In cognitive load theory, "a schema is defined as a cognitive construct that permits us to treat multiple elements of information as a single element" (Sweller, 1999, p. 10). A schema can be processed as one element in working memory, regardless of its complexity (van Merriënboer & Sweller, 2005). Therefore, what comprises one element for an expert may consist of many more elements for a novice, due to differences in available schemas (Kalyuga, 2008; van Merriënboer & Sweller, 2005). For example, an expert chess player may treat an entire board configuration as a single element – a schema; while a novice sees each piece as an individual element. Hence, learners of different levels of expertise may encounter different levels of element interactivity in identical tasks.

2.5.2 Forms of Cognitive Load – Extraneous and Germane

There are two further forms of cognitive load, *extraneous* and *germane*. Extraneous cognitive load refers to the harmful load that does not assist in schema acquisition, while germane cognitive load refers to the beneficial load that is essential in schema construction while learning (DeLeeuw & Mayer, 2008; Kirschner, 2002; Sweller, 1999). However, not all authors agree that these forms are sufficiently differentiated or clearly defined (Moreno, 2006). While these three kinds of cognitive load, intrinsic, extraneous and germane, have proved sufficient to account for research results, there are arguments that a conception of *meta-cognitive load* would enrich the theory (Valcke, 2002). Indeed, there exists evidence that meta-cognition plays a part in deciding when cognitive overload has occurred and what should be done about it (Clark, 2001).

2.5.3 Cognitive Load Theory – "effects"

Traditionally, cognitive load theory has dealt with ways in which extraneous cognitive load can be reduced. This has resulted in a number of *"effects"*: phenomena that, when accounted for by instructional designers, enable more efficient learning to take place (see van Merriënboer & Sweller, 2005). A wide body of literature confirms the existence of these effects and the conditions under which they are applicable (Jamet & Le Bohec, 2007; Kalyuga, Chandler, Tuovinen & Sweller, 2001; Kalyuga, Chandler & Sweller, 1998; Kalyuga, Chandler & Sweller, 1999; Tarmizi & Sweller, 1988; Tindall-Ford & Sweller, 2006; Tuovinen & Sweller, 1999). These studies note that cognitive load theory effects are generally only present when the level of interactivity amongst the problem elements is high. This is because when element interactivity is low, there are already sufficient free cognitive resources to deal with the information presented, and therefore learning will take place regardless of any interventions.

For novices, the additional load in a high element interactivity problem comes from having to hold elements of knowledge *and* their relationships simultaneously in working memory (Sweller, 1999). However, as information and relationships become clear and automated, these same relationships which initially added to the cognitive load now permit information to coalesce into a large super-schema that ultimately reduces cognitive load as expertise develops.

2.5.4 Learning in Cognitive Load Theory

A fundamental assumption of cognitive load theory is that learning occurs as a person engages in learning experiences through which information is encoded into long-term memory. This information is categorised and associated with existing schemas by the learner and no learning occurs unless there is a change to the schemas in long-term memory (Schnotz & Kürschner, 2007). As these schemas are repeatedly used, they are reinforced; each time they are used, they become more highly automated. The more automated these schemas become, the less cognitive resources they consume, and thus, the more cognitive resources are available to other learning or problem solving tasks (Carlson, Chandler & Sweller, 2003; Clark & Feldon, 2005; Schneider & Shiffrin, 1977; Shiffrin & Schneider, 1977). Cognitive load theory's view of working memory differs functionally from Baddeley's (2001) model in that the central executive's function comes from schemas stored in long-term memory rather than a separate central executive. The theory argues that schemas contain information about how thinking about that schema should proceed and as such, act like a domain-specific central executive (Sweller, 2005; Schnotz & Kürschner, 2007). Current views based on brain imaging research and impairment to function of those suffering brain trauma acknowledge the role of the pre-frontal lobes in directing attention and carrying out executive function (Engle & Oransky, 1999; Squire & Kandel, 1999).

2.6 Constructivist Views of Learning

Learning, however, has multiple theoretical conceptualisations. Of these conceptualisations, constructivism in its various forms has proved influential. Constructivists argue that learning takes place through the creation and refinement of schemas (Krause, Bochner & Duchesne, 2003). Students makes sense of (create meaning for) new experiences and new information through the lens of prior experience. This view is very similar to that held by proponents of cognitive load theory. Constructivists (in particular, dialectical constructivists) emphasise the social aspects of learning, such as the learner's educational environment, influences, instructors and experiences (Bruning et al., 2004). The sense-making process is covert and teachers do not have direct control of it – they can only influence it through the aforementioned social aspects of learning. One notion that is important to constructivist views of learning is the Zone of Proximal Development (ZPD). The ZPD refers to the area between the difficulty of problems that students can solve on their own and that they can solve with guided assistance (Belmont, 1989; Bruning et al., 2004). Since the lower boundary of the ZPD shows what they can do alone and the upper boundary shows what they can do with help, the ZPD is also representative of the potential ability level of the student (Belmont, 1989). The upper level of ZPD will vary according to who the guider is (Belmont, 1989), which demonstrates that the ZPD, and therefore the potential ability of students, is socially governed in part.

2.6.1 Scaffolding

The process by which an expert guider and a novice learner collaborate in problem solving, with the expert's aim of providing minimal but precise assistance at critical points in the solution path, is known as scaffolding. Scaffolding has been shown to be an effective means of improving learning and achievement (Beed, Hawkins & Roller, 1991; Bruning et al., 2004;

Hattie, 2009). Scaffolding is also a factor in transfer of general problem solving skills to new problem areas, with Klaczynski (1993, p. 679) stating that "a number of studies have shown that transfer is unlikely unless explicit clues are provided that point reasoners to similarities between problems".

There are many forms that scaffolding can take; however, one kind of scaffolding that an expert can give is planning support. The study by Delaney, Ericsson and Knowles (2004) examined the role of planning in problem solving. They discovered that a group that planned a solution path before solving a problem had greater solution accuracy than a control group. The planning group's solution paths were also closer to the optimum solution path. This study, however, did not compare the effectiveness of planning with the effectiveness of any other means of scaffolding. In the Delaney et al. (2004) study, a problem lacking in easily definable sub-goals was specifically chosen. This is unfortunate, because the problems encountered in the present study depend on sub-goal problem solving. Thus, while its results are still relevant, they cannot be directly correlated with the current study.

2.7 Problem Solving in Gifted, High-Ability, Expert and Novice Students

Cognitive load theory originally arose from research into problem solving. It has been consistently found that experts work towards a solution by using superior, schema-based strategies; while novices work back from the solution using means-ends analysis (Anderson, 1981; Gorodetsky & Klavir, 2003; Owen & Sweller, 1985; Sternberg, 1998; Sweller, 1999). While this finding is widely accepted, no cognitive load studies that were accessed contained any references to gifted students or students of above-average ability. There is evidence that gifted students solve problems in markedly different ways to average learners. Gifted students show faster acquisition of knowledge and superior problem solving abilities, which makes the

comparison between gifted and expert a natural one (Gorodetsky & Klavir, 2003). It has been suggested that while experts have expertise in their specific domain, the gifted also have expertise in the general domain of problem solving (Sternberg, 1998, as cited by Gorodetsky & Klavir, 2003). Many parallels can be drawn between gifted and expert problem solving, including: (a) better encoding of solution-relevant information; (b) accurate linkage of prior knowledge with current-problem information; (c) proficient analogical reasoning allowing them to link problems by their deep structure, even when surface structures are different; (d) efficient translation of problems into terms understood by the solver; and (e) solution processes which are goal directed (Gorodetsky & Klavir, 2003; Sternberg, 1998). Sternberg (1998) attributes many aspects of expertise to metacognitive functioning.

2.7.1 Self-efficacy

There are many other areas influencing task performance in gifted students. In the area of mathematics, Hong and Aqui (2004) have identified that giftedness positively influences individuals' mathematical self-efficacy, perception of ability and the value they placed on learning mathematics. Montague and Applegate (2000) also attribute task performance to self-perceptions of ability, academic competence and task difficulty. Pajares (1996) found that gifted students had higher self-efficacy in mathematics and self-regulated learning, and lower mathematics anxiety than normal students. Further, mathematical self-efficacy was significantly correlated with problem solving performance. This result is in accordance with Bandura's (1997, as cited by Pajares, 1996) predictions. Interestingly, cognitive ability was not a direct predictor of problem solving performance in the gifted; however, it was a direct predictor in normal ability students. In gifted students, its effects were mediated by prior achievement and self-efficacy. This counter-intuitive result serves to highlight the crucial role of self-efficacy in mathematical problem solving.

2.7.2 Creative Thinking and Motivation

The ability to think creatively also facilitates solutions in open-ended problems. Gifted students use their creative thinking more often in these sorts of situations (Russo, 2004). However, the Russo (2004) study also found that gifted students' motivation level was influenced by problem type. Only if the problem was of interest to the student did they "approach it with considerable enthusiasm" (Russo, 2004, p.185). In problems where gifted students were not highly motivated, their performance dropped to a level close to that of the normal-IQ group (Russo, 2004). While this was not a study of motivation, these results confirm the belief that motivation is a strong predictor of task performance (Stipek, 2002).

2.7.3 Schema-based Instruction

A study that illustrates many relevant points of problem solving from a schema theory perspective was undertaken by Jitendra, Griffin, Haria, Leh, Adams and Kaduvettoor (2007). This study used a quantitative approach to determine the effectiveness of schema-based instruction as compared to general strategy instruction in 88 3rd-grade students. According to Jitendra et al. (2007), "[Schema-based instruction] explicitly analyzes the problem schema ... and the links pertaining to how the different elements ... of the schema are related." No distinction was made between expert and novice problem solvers; however, students were pretested and post-tested and re-tested six weeks later to evaluate maintenance effects. They found that schema-based instruction was more effective than general strategy instruction in terms of enhancing students' mathematical word-problem solving skills and generating transfer of knowledge skills. However, a limitation of this study was that schema-based lessons were taught in groups of 15 students at a time for 25 minutes. Any opportunity for one-on-one instruction was limited to that normally available within a normal classroom setting. An

additional caveat of the study was that it was conducted in an area of very low socio-economic status. These two points are in contrast with the current study's focus of one-on-one interactions in private high schools. However, the strong focus of schema-based instruction in the Jitendra et al. (2007) study makes it closely connected to the current study.

2.7.4 Verbalisation

Verbalisation during problem solution has been found to affect student performance in students who performed above the average in mathematics but below the average in numeral base systems (Zook & Di Vesta, 1989). In the Zook and Di Vesta (1989) study, students were told to either verbalise or to think about the reasons for their choice of an equation to solve a numeral base system problem. The study compared the effectiveness of overt verbalisation (the "tell me" group) to covert verbalisation (essentially, guiding the students about what to think about – the "think about" group). Zook and Di Vesta (1989) found that the overt verbalisation group worked forward with less excess moves than covert verbalisation group, but at the cost of a slower solution time. Overt verbalisers were also slightly better at simple transfer problems but no difference was found for complex transfer problems.

One shortcoming of the Zook and Di Vesta (1989) study was that it was a strictly controlled experimental study. Subjects read the instructions from instructional packets and the role of the experimenter was very limited. While keeping the experimenter distant may have helped Zook and Di Vesta reach their research goals, having a distant experimenter diminished the natural feel of the experiment and prevented the researcher and subject from interacting freely. Further, it tended to prevent the exploration of any cognitive phenomena apart from that which was being studied. There was no opportunity to 'seize the moment'. These limitations are not present in the non-linear, spontaneous interview style of the current study.

The Zook and Di Vesta (1989) study's method did, however, minimise the possibility that particular individuals can significantly skew results.

Finally, the Zook and Di Vesta (1989) study worked with students who were novices in the problem area. Experts might be expected to perform differently, as overt verbalisation would not be expected to give experts an advantage at solving a problem they are already familiar with. Zook and Di Vesta's (1989) experiment is nevertheless valuable as it demonstrates that overt verbalisation has an effect on problem solving abilities, increasing the use of forward working, schema-based strategies as opposed to means-ends analysis. Given that overt verbalisation is a central characteristic of the interview stage of the present study, this result should be taken into account.

In a study by Montague and Applegate (1993), it was found that gifted students made significantly more metacognitive verbalisations than average intelligence and learning disabled students as problems became more difficult. However, for simple problems, no difference was obtained. This suggests that the difference in cognitive capabilities between gifted and other students becomes more pronounced as cognitive load increases. It must be noted that although gifted students were a category of participant in this study, its focus was on students with a learning disability.

2.7.5 Gender Differences in Psychological Aspects of Problem Solving

Preckel, Goetz, Pekrun and Kleine (2008) considered achievement, self-concept, interest and motivation in mathematics in male and female gifted and non-gifted grade 6 students. It was found that gender differences in these areas are increased in the gifted population. Gifted

boys were found to have a significantly higher mathematical self-concept and interest level than non-gifted boys. Mastery or performance goal orientation did not differ significantly between any of the groups. However, this study used only non-verbal reasoning ability to separate gifted and average-ability students. As this study has a mix of male and female participants of high mathematical ability, this result should be considered in the analysis of the present study.

Similarly, Randhawa, Beamer and Lundberg (1993) considered the role of self-efficacy in mathematics achievement. It was found that mathematical self-efficacy had a strong role in mediating achievement. Attitudes towards mathematics were not found to have a significant effect on achievement. The authors also reported that the effects were much greater in strength for boys than for girls. This is alarming given that girls, on average, have lower mathematical self-efficacy than boys (Randhawa et al., 1993). However, regardless of gender, self-efficacy has an experimentally verified effect on mathematical performance.

2.7.6 Transfer of schema-based knowledge to new areas

Novick (1988) undertook research on analogical transfer using expert and novice problem solvers. In this study, analogical transfer was conceptually the same as using the solution path of previously solved problems to solve new problems. This idea is fundamentally representative of using schema-based problem solving methods, although this connection was not made in the paper (see Novick, 1988, p. 510). Students who used analogical transfer could be said to be applying previously created problem solving schemas to a new area. Some degree of expertise was required by students so that they selected the appropriate previous schema based on structural and not surface aspects of the newly presented problem. It was found that experts could use prior knowledge based on its surface and structural content,

while novices tended to rely only on surface similarities. Other studies have also found that IQ and strategy transfer are strongly linked (Klaczynski, 1993; Klaczynski & Laipple, 1993). Higher successful transfer levels show a more developed understanding of a given schema, as is evidenced by its use outside a familiar context.

2.7.7 Internal Representations

Another area of inquiry into gifted problem solving is the area of internal representations. Research suggests that internal representations of problems can take the form of (a) learners paraphrasing problems; (b) visualising problems through written or internal diagrammatic representations; and (c) goal setting and planning solution strategies (Montague & Applegate, 2000). Montague and Applegate (2000) found that gifted students solved problems significantly quicker than non-gifted students but did not differ in the amount of problem solving strategies used or perceived difficulty of the problem. Presumably, this is because the problems were not difficult enough to require additional problem solving strategies. This suggests that gifted students have clearer internal representations of problem forming schemas that are more highly automated than those of non-gifted students. It must be noted that although gifted students were participants in this study, its focus was on students with a learning disability.

2.7.8 Problem Solving Approaches

When cognitive load theory was still in its infancy, Owen and Sweller (1985) conducted a study investigating what students learn while solving mathematics problems. They conducted an experiment where twenty 10th-grade students were tested individually to ascertain whether means-ends analysis retards schema acquisition. Some students in a control group were left to work backwards using means-ends analysis while others were forced to work forward by the

use of goal-free problems (see Owen & Sweller, 1985; Sweller, 1999, for a description of goal-free problems). It was found that the forward workers in the goal-free problem group showed a higher degree of success in solving the given problems more successfully than the control group. Two additional experiments were carried out, further showing the advantages of goal-free problems above traditional problems in mathematics instruction.

While Owen and Sweller (1985) worked with students individually, the essence of the study was still quantitative. The researchers' choice of methods bound the degree of freedom allowed in the interactions between themselves and their subjects. Further, the conclusions reached from this study, undertaken over 23 years ago, are no longer cutting-edge knowledge in cognitive load theory. While the method of Owen and Sweller's (1985) research is closer to the proposed study than most of the current cognitive load theory studies, it has been a long time since a truly exploratory study into the effects of cognitive overload has been undertaken (that is, if it ever has been). Clearly, it is time to delve more deeply into the effects of cognitive load during problem solving.

2.7.9 Qualitative Research on Problem Solving Experiences

Sriraman (2003) conducted a qualitative study with a grounded theory analysis on nine students' mathematical problem solving experiences. Four of these students were categorised gifted and five were non-gifted. This study contained a first stage where students formed responses to given problems in a diary. In the diary, questions were asked related to how learners paraphrased problems and planned solution strategies. These questions were designed to access the students' internal representations, as suggested by Montague and Applegate (2000). In a second stage, students were interviewed concerning their diaries. As Sriraman (2003, p. 155) states, "these [interview] questions were formulated with the hope of

enabling students to verbalise their solution strategies". The proposed study has the advantage over Sriraman's (2003) study that the interview questions will enable students to verbalise their solution strategies as they are taking place. This eliminates any problems with remembering solution strategies that may have been present in Sriraman's (2003) study, given that his interview stage took place at least a month after the first problem was initially solved. The current study should generate a feeling of immediacy and spontaneity that was not present in the Sriraman (2003) study.

Another aspect of Sriraman's (2003) study that may have had an effect on results was that the researcher was also the class teacher at the time. However, the researcher used a colleague to analyse random slices of data as a test of reliability. This resulted in approximately 90% agreement between the researcher and the colleague. Regardless, the way in which having the researcher as the class teacher may have affected the results is unknown at this time. The proposed study will have the researcher as an external entity, which may add objectivity to the results.

Sriraman's (2003) study found that only the four gifted students discovered a generalisation in the questions, while the five non-gifted students did not. This finding agrees with research stating that gifted problem solvers can access the deep structure of a problem more effectively (Gorodetsky & Klavir, 2003; Sternberg, 1998). The gifted students also displayed an emotional response to finding a generalised solution for the problems, which agreed with findings in the literature (von Glasersfeld, 1987, as cited by Sriraman, 2003). This emotion was a strong motivator in these students. Sriraman (2003, p. 163) found that this strong motivation led to "remarkable perseverance" in these students – a result also reported by Russo (2004). These findings are consistent with current research indicating that upon experiencing a

cognitive breakthrough, the brain releases endorphins that are detected by the brain's opiate pleasure receptors. This acts as a reward system which ensures motivation for further problem-related activities (Biederman & Vessel, 2006; Greive & de Berg, 2008).

2.8 Rationale for the Present Study

While many studies have been conducted in gifted or high ability students' mathematical problem solving, and in cognitive load theory, our understanding of the two is far from complete. None of the studies conducted on mathematical problem solving in gifted or high ability students use concepts from cognitive load theory to explain their results. Similarly, none of the studies conducted on cognitive load theory use gifted or high ability students as an explicit category of participants (granted; they do use experts in certain areas). The two areas are not incompatible. Thus, there is an opportunity to conduct research on an area that has not been studied, using a qualitative methodology that is rarely used in research undertaken from a cognitive load theory perspective.

2.9 Conclusion

Cognitive load theory was born from problem solving research. It now has an opportunity to give something back to problem solving research. Van Merriënboer and Sweller (2005, p. 165) describe their view on the future direction that cognitive load theory research should take: "Future [cognitive load theory] research should provide a more detailed description of how cognitive structures develop with expertise, how they enable learners to adapt increasingly to task demands..., and how they relate to improvements in self-regulated learning." This research aims to contribute to fulfilling these needs by conducting a comprehensive qualitative enquiry into the problem solving processes of students with high mathematical abilities, and aims to describe in detail the way that these students respond to changes in

cognitive load throughout the problem solving process.

Chapter 3

Method

3.1 Introduction

This chapter outlines the research design employed in this study. It discusses the method of data collection, the test questions used, and describes the analysis of data. It also outlines the ethical issues taken into consideration during the implementation of the method.

3.2 Background to the Study

The study was conducted in two independent high schools and focused on three students from each school (see table 1). Participants were supplied with code-names. The first school had two year-seven participants and one year-eight participant, and the second had three year-eight participants. Of the participants from the first school, two were male and one was female; of the second, one was male and two were female. The participants from the first school were selected from the top mathematics class. The participants from the second school were selected from the second top class. It must be noted that while these participants were of high mathematical ability, there was no evidence to suggest they were gifted students. The participants took part in several problem solving or interview sessions. These consisted of a Raven's Standard Progressive Matrices test, a geometry problem booklet, an algebra problem booklet, and a final interview session. The geometry and algebra problem booklets were created by the researcher and were checked by a mathematics teacher and the present study's supervisor for validity. All the relevant questions are included in Appendix A and B. Participants were asked to verbalise their cognitive processes while they were solving the

problems. During or after the question, there was a discussion where the participants answered questions posed by the interviewer. During each of the sections, a digital audio recording device was used to record the conversation between the participants and interviewer. There were 16 interview sessions in total. The interview sessions ranged from about 20 minutes to 65 minutes, with most around the 45 minute mark. After every session, informal field notes were included outlining important observations arising from each session along with any other thoughts I may have had.

The Raven's Standard Progressive Matrices test was administered as a measure of general fluid intelligence (see table 1). Each participant completed 48 questions (sets B to E, 12 questions per set) within a tight 30 minute time limit. The teachers for each group of participants also provided a list of marks and comments for each participant. The marks for the first school are half-yearly report marks (a combination of assessment marks for the first half of the year). The marks for the second school are the mean of the marks provided for 7 tests so far this year. Hannah was absent from the 3 most recent tests. Wendy was absent from one test. There was no extra weighting given to any test in computing the mean mark. The marks between the two schools are not comparable; however, the Raven's Standard Progressive Matrices marks are.

Table 3.1: Participant information table

	Grade	Gender	School	RPM	Combined Test Marks	Teacher's comments
Matthew	8	M	1	77%	87% (Rank: 2/27)	Matthew is an exceptional mathematician when he can be bothered. He has good problem-solving skills; however he often tends to think he can solve a problem before he tries to solve it. For this reason, he is a little apathetic about improving on his basic mathematical skills and communication skills. When he has the necessary skills for a particular problem, he is almost always able to come to a correct conclusion.
Thomas	7	M	1	77%	72% (Rank: 10/27)	Thomas is quite a capable mathematician. He shows good understanding of concepts and, when encouraged, he produces excellent solutions. He solves problems well but is not good at communicating how he arrives at the answer. He doesn't like to stand out and he can be a little lazy so he often works at a standard below that which he is capable of.
Rosie	7	F	1	75%	70% (Rank: 14/27)	Rosie is often slow to grasp a concept initially, but is able to solve most problems with a little assistance. She does very well in familiar contexts and has a reasonable grounding in basic mathematical skills to help her.
Jake	8	M	2	79%	75%*	Jake is a sound mathematics student with a moderate to above average ability. He is confident in asking questions but often makes simple mistakes or misses important points in concepts and needs one-on-one assistance to gain confidence. He has a good ability to communicate what he is processing.
Wendy	8	F	2	69%	77%*	Wendy is a strong mathematics student with a high ability. She is sometimes over confident in topics but with other topics she lacks confidence. She has a good ability to communicate what she is processing.
Hannah	8	F	2	71%	76%*	Hannah is a sound mathematics student with a moderate to above average ability. She often lacks confidence in Maths until she receives one on one tuition and encouragement. She has a sound ability to communicate what she is processing.

* Rank not provided by the teacher of school 2.

The transcripts and field notes were subjected to thematic analysis (for a sample of coding categories, see Appendix C). The purpose was to explore the effects of cognitive load on higher ability students as they solve mathematics problems. As this was strictly an exploratory study, research hypotheses and specific research questions guided the study rather than constraining its direction. Instead, the data was allowed to determine the direction and conclusions that were made.

3.3 Aims of this study

This study aimed to investigate the interactions between cognitive load and mathematical problem solving in high ability mathematics students. The goal of this study was to collect as much information as possible about these interactions, and then analyse it and report it in a way that would generate a descriptive and explanatory theory or model about the problem solving process, with cognitive load as the central focus.

3.3.1 Research Question

With those aims in mind, the following research questions were proposed:

1. Can cognitive load theory be used to explain observations of student problem solving performance?
2. If so, can a model incorporating cognitive load theory be created to guide teaching for problem solving?
3. What implications does the model hold for teachers?
4. What implications does the model hold for students?

3.4 Ethical Procedures and Protocols

This project was designed to be as unobtrusive as possible. To this end, teachers were consulted to find the most appropriate time and place to meet with the students. The teachers were asked about the topics they were teaching in class and the appropriate type of question (geometry or algebra) was given either during the time or just after the topic had been taught in class. This allowed the problem solving sessions to act as a challenge activity for the participant, where they would encounter questions that were more difficult than usual. The fact that they were helped through these questions allowed the participants to benefit from these sessions in a similar way than they would have benefitted from coaching. Further, I gave each participant tips after each session about how they could organise their thinking to reduce their cognitive load in problem solving. I gave them strategies and techniques that could be applied to any kind of problem. Thus, the benefits of the study were beyond even the realms of geometry and algebra.

The study received ethical clearance from the Avondale College Human Research Ethics Committee and permission to conduct the study was obtained from the principals of each school, the teachers of each class involved, the parents of each participant and the participants themselves. Student participation was voluntary and withdrawal from the study without penalty was allowed at any time. Potential participants were identified by the class teacher. A student information sheet requesting consent was given to each participant (see Appendix D). Each participant's parents also received a parental information sheet and consent form (see Appendix E). All names of participants, teachers, schools and any other participants in this study were kept confidential. All research data will be kept according to the provisions outlined in the ethics approval forms and destroyed after the appropriate time interval.

3.5 Research Procedures

In sessions one and two, scaffolding techniques were be used to facilitate participant achievement. If there were any parts of the problem where the participant could not independently proceed forward under any circumstances, scaffolding was used to facilitate a solution. This allowed the monitoring of cognitive load throughout the problem solving process without ending any problems prematurely due to non-solution.

3.5.1 Session One – Raven's Matrices

The first session was centred on the Raven's Progressive Matrices test; a test which employs visual-spatial pattern recognition to give a measure of generalised fluid intelligence. Both groups of participants were given half an hour to silently complete sets B, C, D and E. This ensured the four most difficult sections of the test were attempted. The participants' answers were written on an answer sheet and were marked after the session from the official answer sheet. For both schools, this session took place with all three participants in the same room at the same time. Participants were given a brief opportunity to share some strategies they used to solve the problems, along with sharing any thoughts they may have had about the session. All participants were present for session one.

3.5.2 Session Two – Geometry

The second session had each of the six participants doing a paper on geometry. Each participant did this session individually. Participants first solved the question while verbalising their thoughts. After solution, questions were asked of the participants relating to specific moments in their problem solving process and cognitive load throughout. These were asked

after solution had taken place to ensure minimal interference with fluency of thinking. Scaffolding and assistance was given if required (if little progress had been made, the participant appeared unable to continue, or if help was asked for). The paper contained 13 questions but only one participant completed 9 questions. All of the other participants completed 8 questions. One question was designated as the "overload question", which purposely contained much more information elements than the other questions, in an attempt to induce cognitive overload. I spent more time on this question than any of the others, both with the solution path and the interview afterwards. For the question booklets used by the participants in the geometry session, see Appendix A.

After all the problems were solved, the participants had an opportunity to make any additional comments. Immediately after the end of the session, field notes were made of any points considered important. All participants completed this session.

3.5.3 Session Three – Algebra

The third session had only five of the six participants (Hannah was absent) doing a paper on geometry. Again, each participant did this session individually. The participants first solved the question while verbalising their thoughts. After solution, the participant and I spoke regarding specific moments in their problem solving process and cognitive load throughout. Scaffolding and assistance was given if required. The paper contained 9 questions and all the participants completed all the questions. Question 9 was designated as the "overload question". For the question booklets used by the participants in the algebra session, see Appendix B.

The final task was for the participants to subjectively rate the cognitive load imposed by each of the questions. The construct of cognitive load was portrayed as "how much stuff was in your head; how much information you needed to think about at once." This was ranked on a scale from 1 being the least possible load to 10 being the greatest possible load. They also retrospectively rated their cognitive load on the geometry questions. The only participant who did not rate their cognitive load on either of the tests was Hannah, due to absence. Participants again had an opportunity to make any additional comments or discuss any questions. Immediately after the end of the session, field notes of the most important parts of the session were made. Hannah was absent for this session so she did not attempt it.

3.5.4 Session 4 – Interview

All six participants were involved in a final interview session. No problem solving was involved in this session. Participants were interviewed in a group of three at School 2, however only Matthew and Rosie were interviewed at School 1; Thomas was interviewed over the telephone. For the list of questions that were planned to be asked during the interview session, see Appendix F. Of course, given the spontaneous nature of interview sessions, the interview sessions usually deviated significantly from these questions in certain sections. Participants interacted and dialogued freely in this session, result in some valuable exchanges between participants.

3.6 Data Analysis

The Raven's results, audio interviews, field notes, answer sheets, student assessment marks and teachers' comments were collected. A set of coding categories was constructed and I

went through the transcripts, flagging themes that matched up to the pre-defined categories. The categories were initially created based on my overall impression of what had come out of the sessions most prominently. As the coding process went on, it became apparent that the original categories needed to be refined. The coding categories were modified twice while the coding process was going. After this coding was completed, I related all of the categories into a graphical representation or model. Participants were asked to provide a subjective rating of the cognitive load imposed by each question on a ten-point scale. These ratings allowed me to see which questions participants found the most difficult, and which participant had the most difficulty with each of the sessions (see Appendix G). After a process of writing the results chapter, going back over the transcripts and noting any comments in the field notes for their implications, the model was refined to its current state (see figure 4.6, in §4.8). The data that led to the creation of the proposed model outlining problem solving processes in these participants will be examined in detail in the results chapter. The model will be presented in the discussion of the results.

Chapter 4

Results

4.1 Introduction

This chapter examines the problem solving process using the collected data as evidence. The central construct that is referred to throughout this section is that of cognitive load. According to Cooper (1998, §4.1), "cognitive load refers to the total amount of mental activity imposed on working memory at an instance in time." This total amount of mental activity can also be thought of as representing total amount of cognitive resources. These resources, which are limited in capacity, can be occupied by problem elements, metacognition intentions and strategies related to potential solution paths. Rather than focusing specifically on each of the three forms of cognitive load, a general view of cognitive load will be taken unless it is necessary to refer to a specific kind of cognitive load. Although it is impossible to quantitatively measure instantaneous cognitive load directly, especially given the nature of this study, qualitative approximations based on the observable behaviours of the participants, or the participants' own subjective assessments, will be used.

This chapter presents three facets of problem solving: (a) reading, internalisation and classification, (b) solution strategies, and (c) problem restructuring strategies. Having established the importance of a manageable amount of cognitive load in arriving at an effective solution path, the chapter documents the process of the solver as they traverse the solution path, giving due reference to cognitive load issues where necessary. Finally, the chapter documents the ways in which the participants gave evidence of the use of load-reducing techniques in their problem solving.

4.2 Reading, Internalisation and Classification

The first thing a problem solver must do upon being given a problem is to read, internalise (create an internal representation) and classify it. Reading allows the problem solver to become acquainted with what is being asked of them. Classification then takes what has been read and asks the question, "Does this match up to anything I've seen or done before?" If the answer is yes, then the solver can immediately relate the current question to a schema held in long-term memory. Sometimes, the problem may require restructuring in order for this relationship to become apparent. Relating the question to a schema immediately puts the wealth of prior knowledge available in this schema at the solver's fingertips. As participant, Matthew says, "Do you know how there's different, like, types [of problem]? Like there's algebra, there's perimeter, there's area, yeah. Whatever type, there's always a strategy that you use for that" (Interview Session). This quote neatly shows the process of reading a question, internalising and classifying it into a certain type, and using strategies for that type of question as governed by the schemas that exist in long-term memory to locate a solution. The richer this schema is, in terms of the amount of knowledge it contains and the level of automation it has attained, the more powerful its effect will be on the problem for which it is used. Of course, not every person will have a well developed schema for every problem type. The rest of this chapter will explore some of the ways in which students with rich and poor schemas solve various kinds of mathematics problems.

4.2.1 Reading, internalisation and classification – easy questions

The reading, internalisation and classification style that was displayed by the participants was different for high and low difficulty questions. At this point, it is beneficial to remind the

reader that problem difficulty arises from the intrinsic cognitive load in a problem *and also* from the amount of prior knowledge in the solver's long-term memory. A person with rich, automated schemas in a certain area will be able to solve a problem with considerable ease as compared to a person with poor or non-existent schemas and, consequently, consider the problem easier. Relating to low difficulty questions, participant, Wendy, states, "If it's like, really simple, I'll just look at it and understand it and just write down what I've learnt. So, like, what I've learnt to do with it." (Interview Session). This shows that for a standard question, whose form has been met before, Wendy's schema in long-term memory tells her how to solve that particular type of question. This schema is activated upon reading the question and guides her through the solution path until the solution is achieved. In this case, the required cognitive resources lie within the capacity of working memory. Similarly for participant, Hannah, "For me it kind of just says how I should do it, just in the question itself ... automatically" (Interview Session). This shows that there is an oversight of the question, where the solution path is immediately linked to the identified problem type and the solution comes with complete clarity. Clearly, there is very little cognitive load involved in solving this type of question.

4.2.2 Reading, internalisation and classification – difficult questions

For more difficult questions, however, the way in which reading, internalisation and classification takes place is different. Thomas, a participant, gives a detailed recount of some strategies he uses to solve difficult problems.

Thomas: Maybe [I] simplify [the problem] into different questions, like change it around ... or maybe think back through the other questions [that I have done in the

past]... or halve the numbers ... try and put down all my information and work it out straight away and if I'm not right, I'll just leave it' (Interview Session).

While these particular strategies will be discussed later in this chapter, from a cursory glance it can be seen that this kind of question requires substantially more complex processing in order to solve. It will be argued that each of the strategies that Thomas has employed is a way of reducing cognitive load, and that this reduction of cognitive load is what allows the solver to progress towards the solution. There is no indication of an automatic or even clear solution path here; the solver needs to work hard to produce a solution. For Wendy, this difficulty goes even further than Thomas':

Wendy: Sometimes I'm reading it, and I read it, and I read it, and read it; it doesn't really come to me. Like, sometimes I'm just reading it but I'm not actually taking it in...

Hannah: Yeah.

Wendy: So I just give up. (Interview Session)

Continuously re-reading a problem without restructuring it means the problem type is not identified, there is no understanding and the internal representation is fragmented. The cognitive load imposed by the problem is so great that cognitive clarity cannot be achieved by the participant if the question remains in its current state.

With respect to processing the large amount of information in difficult questions, Hannah states, "Yeah, like the angles; umm, you're not sure how you can, like, relate it to all the other ones." (Interview Session). This example, which is characteristic of cognitive overload, shows

the cognitive stress that questions with high element interactivity[2] can place on a problem solver (see also van Merriënboer & Sweller, 2005, p. 149). This example shows unresolved element interactivity. Hannah has been unable to establish the nature of the relationships between the disparate elements and so has not been able to form an internal representation. In contrast, resolved element interactivity produces a single chunk of information instead of a number of elements that, when unrelated to each other, cause overload. When the participants were faced with a question of high cognitive load, certain strategies were used to manage the amount of cognitive load.

4.3 Solution Strategies

This section will discuss the most prevalent solution strategies that the participants used. These are:

1) Finding and writing down relevant problem information

2) Getting rid of irrelevant, distracting information

3) Splitting up a problem into smaller sub-problems

4) Transforming a problem state into a different form

5) Recognising patterns within the problem.

The first three are information management strategies while the last two are problem restructuring strategies.

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[2] Sweller (1999), and Sweller and Chandler (1994) explain that as element interactivity increases, the more difficult it is to consider individual problem elements in isolation. Thus, it is more difficult to reduce cognitive load by breaking material up into smaller pieces – in essence, the learner is *forced* to consider all of the elements simultaneously. This leads to increased cognitive load. The same argument can be extended to element interactivity in problem solving – students are unable to break a question up into smaller elements because each element interacts with the others.

4.3.1 Information Management Strategies

For the three information management strategies, I will examine the process that Matthew undertook in solving Question 9 of Session 2 of the algebra problem booklet (see figure 4.1).

9. A boy adds together the number of PS3 games and Xbox 360 games he has. He has 8 games in total. He then multiplies the number of PS3 and Xbox 360 games he has and finds that they multiply to 12. How many of each kind of game does he have? He has 4 more PS3 games than he has Xbox games.

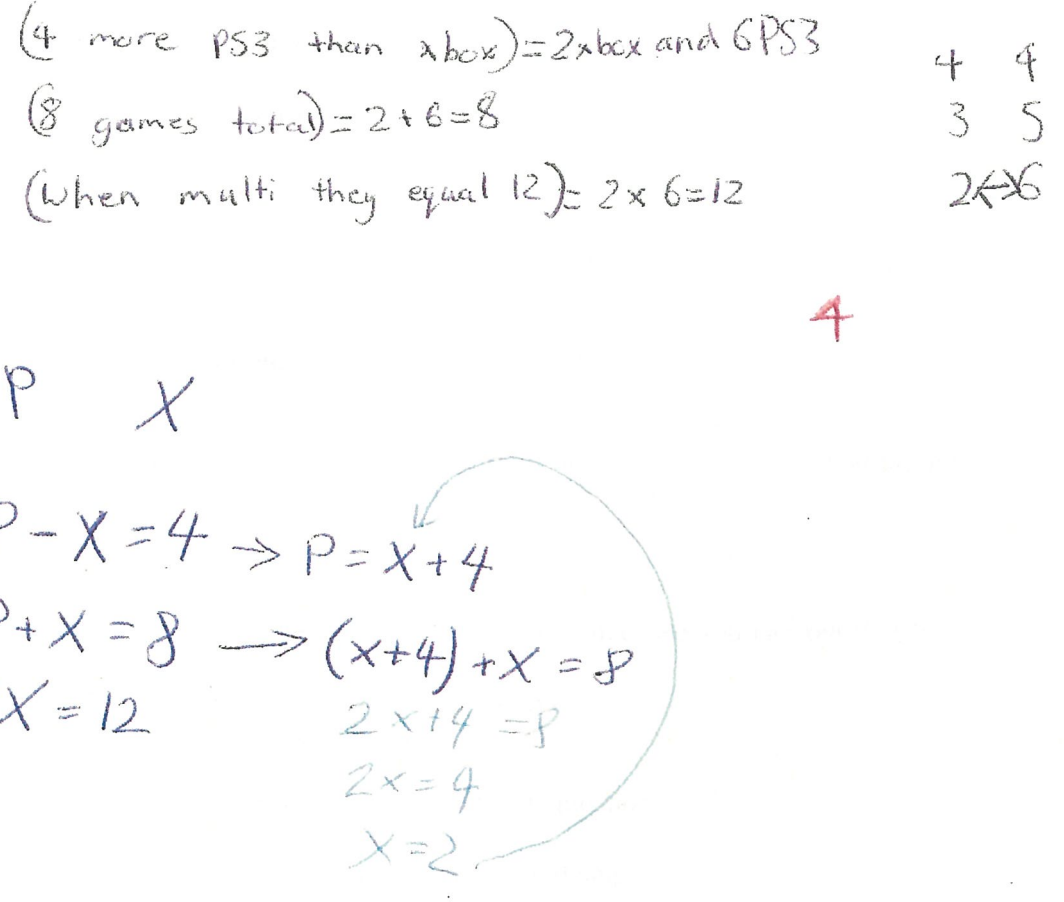


Figure 4.1: Matthew – Session 2 (Algebra), Question 9. The bottom part, below the "P" and "X" is my handwriting. This was done after Matthew finished the question to explain simultaneous equations. The red number "4" is Matthew's subjective cognitive load rating for the question, out of 10.

First, Matthew spent about 35 seconds reading the question carefully aloud. Immediately after reading it, he said, "So first, you'd use this part of the question", referring to the final line stating that there are 4 more PS3 games than Xbox games. This was the first piece of evidence that suggested that Matthew broke the problem down into parts. The reason this solution strategy was chosen was because the problem space was simply too large to fit into his working memory as a whole. Matthew had no option but to split the problem up into its constituent parts if he was to avoid cognitive overload. Matthew's next step was to write this line of information in his own words below the question.

Matthew had already extracted information from his first reading of the question; however, this information was not sufficient so enable him to reach a complete solution. Matthew then had no choice but to re-read the question to extract more information from it. After around 1 minute and 45 seconds of trying to solve the problem, the following exchange took place.

Matthew: I'm just struggling to find out what each thing means.

Interviewer: You took [final line of the question] out; why did you take that bit out and re-write it?

Matthew: Because it's the most important thing because you can tell that if you write this down, it'll help later on somehow.

Interviewer: Yep. Is there anything else in the question that you think you could write down the bottom that you think could help you out later on?

Matthew: He has 8 games in total.

This is a subtle form of scaffolding which involves assisting the problem solver with the solution path, however, no information about the problem itself is revealed.

About 20 seconds later, Matthew also wrote down that when the number of games is multiplied together, they equal 12. In the written question, I phrased this sentence somewhat awkwardly to impose an additional extraneous load at that stage. The effect that this had is best stated in Matthew's words.

Matthew: It takes me a while to figure out... because it's in English. You could've wrote it in maths form and I would've got it straight away but because you wrote it in English, I have to think of what it actually means [pause] 'cause English words have more meanings than Maths does.

Matthew indicates here that he has some difficulty internalising word problems and he expended significant cognitive resources on interpreting the individual parts of the question. When he decided on the correct interpretation of each part, he wrote it down underneath the question. From then on, he referred only to the information that he had written down. Matthew had therefore, with some effort and scaffolding, transformed an English sentence with high cognitive load into a 'mathematical sentence' with low cognitive load. Writing it down further decreased the demands on his working memory as he did not have to retain the mathematical sentence in working memory. This freed up cognitive resources that could then be dedicated to solving other parts of the question and eventually, discovering the relationship between the three statements.

After nearly a minute and a half of silence, Matthew read aloud the first two pieces of information that he wrote down. He then wrote down pairs of numbers that add to 8. He continued writing the pairs until he found a pair, "[that had] four difference." While Matthew knew nothing about the method of simultaneous equations, he had found a pair of values that satisfied all three simultaneous equations. After that, finding a solution was simply a case of

using the third equation to tell him which number belonged to which type of game. He arrived at the final and correct answer in 6 minutes, and arrived there with a large degree of certainty. Matthew's problem solving process here was a good example of how difficult problems can be solved effectively by using solution strategies. Although Matthew did not have a schema available to solve simultaneous equations, he was able to break the question up into smaller questions for which he did have schemas. This gave the impression that he was confidently working forward towards the solution.

Throughout the data collection stage, there was a noticeable tendency for the participants to find the most relevant information in the question and write it in their own words in the space reserved for working out. In fact, the *first* strategy of most of the participants tended to be writing the relevant information down beside the question. The goal of this step was to reorganise the information given to the participants in a way that best fit that particular participant. It was found that the participants were most likely to write down information on the longer word questions. There was very little re-writing of easy questions, both in the geometry and algebra sections. This fits in with the theory that writing down relevant information is used as a means of reducing cognitive load. Pen and paper simply act as an extension of working memory. For instance, if there are 8 problem elements but only room for 5 in working memory, it is prudent to write down all the elements and swap them in and out of working memory as the need for processing each element arises.

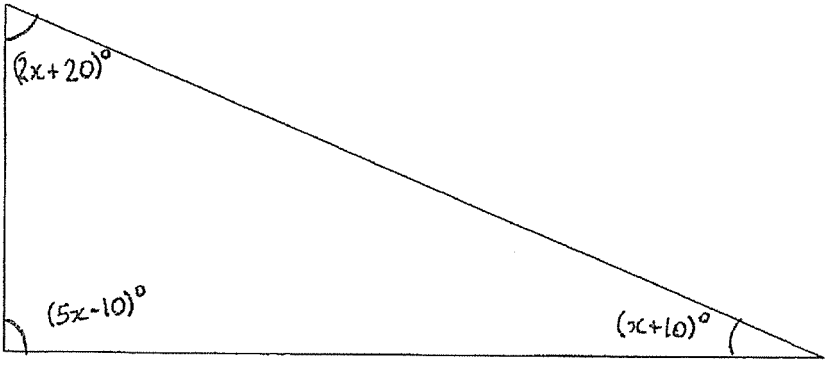
4.3.2 Problem Restructuring Strategies

4.3.2.1. Transformation of problem state

The second group of solution strategies consists of transforming a problem state into a different form and recognising patterns within the problem. In theory, transforming the problem state into another form should have benefits to working memory load. First, it means that key information must be written down, the benefits of which have already been discussed. Second, it allows the participants to reframe or restructure a question according to their preferred form of problem presentation and hence facilitate classification. For example, if a participant is solving a word or trigonometry problem, they can transform it into an algebraic equation. If the participant is proficient with algebra, there is a strong chance that they will be able to solve the resulting equation. This is due to that participant having well developed algebra schemas in long-term memory. Of course, if the participant is not strong at algebra, this restructuring of the problem form may be of no assistance at all.

The following example illustrates problem restructuring strategies. Hannah had been struggling for nearly 2 minutes and 30 seconds on Question 7 of the geometry paper (see figure 4.2).

Name: Date:



7. Find the value of x.

$$8\overline{)160}\quad 20$$

20

Figure 4.2: Hannah – Session 1 (Geometry), Question 7. Only the green handwriting is mine.

After such a long period of difficulty, I decided to find out exactly why she was having trouble.

The following exchange then took place.

Interviewer: Are you finding this one hard?

Hannah: Yeah. Lots of numbers.

Interviewer: That's what makes it hard?

Hannah: I just think, the brackets and adding it all, just with the pronumerals in, like $2x$. So you'd have to find what x is to [pause] times it by 2, I guess. And then plus your 20 degrees. I have no clue what x could be.

Interviewer: So you look at that and there's just too much stuff to look at?

Hannah: Kinda, yeah.

Interviewer: And yet not enough because none of the angles tell you like...

Hannah: Yeah.

Interviewer: ...a number.

This exchange is typical of cognitive confusion, as caused by cognitive overload. Hannah had a basic grasp over what to do (adding the pronumerals, adding 20 degrees), but was unable to organise it in her mind properly. This is evident in the awkward explanation given by Hannah in her second response in this transcript. Realising that Hannah was making no progress and was very confused about what to do, I explained to her that it was an 'angle sum of a triangle' question. This statement activated that schema and the knowledge that comes with it, namely, that all the angles added together equal 180 degrees. I then put it to her that she could represent the fact that the angles add to 180 degrees algebraically by writing an equation, which we formulated together (see figure 4.2, in green writing). After another period of some confusion, I offered the way that I would solve the problem.

Interviewer: What I would do is I'd add this angle – that's all I know about it, that it's $2x + 20$ – then I'd add it to $[5x - 10]$, and then I'd add it to $[x + 10]$, and that's gotta give me 180 degrees. Now, does that give you a clue of what might be the next step?

She thought about it silently for about 40 seconds and then gave me the solution in its entirety. She was able to separate the pronumerals from the number values given, and arrive at $8x + 20 = 180$, and $x = 20$. She did this very tentatively, stating that "This is just random guessing". However, it did permit her to arrive at the correct solution without any further mistakes. When asked to reflect on the problem solving process she had undertaken, she stated:

Hannah: Just setting it out, I think, differently like [an equation] was helpful as well... I think it might have been a bit easier [as an equation, rather than a geometry problem].

Interviewer: Do you know why it would be easier for you?

Hannah: I think just with the angles, looking at them all, it kind of got a bit confusing because there are a lot numbers and you've gotta match them to the angle. But with just numbers in brackets, I think it made it easier.

Interviewer: So they gave you too much information; too much to look at.

Hannah: Yeah, I think so.

This indicates that her difficulty resulted from her inability to restructure the problem in order to reduce cognitive load. When I told her the way that I would solve it, she had trouble dissociating the algebraic equation from the original diagram. This may imply that the participants required 'ownership' of the restructured question. Indeed, on most occasions when given a transformed version of a problem, the participants still tended to refer back to the original problem. There is a natural tendency to do so, but if the original problem was a source of overload, it makes more sense to focus on the new problem state in isolation and ignore the old problem for a moment. An association to the original problem implies an association to the original problem's cognitive load. Holding onto the original representation

diminishes the ability of the new representation to reduce cognitive overload, as the old representation continues to occupy working memory resources. This is particularly true when a schema-based solution to the new problem exists. Once the new question is carefully considered and if required, the participant has the freedom to place the new representation into the context of the original representation, or place the original representation into the context of the new representation.

4.3.2.2 Pattern Recognition

Pattern recognition is vital to problem solving as it allows identification of problem types. A lot of pattern recognition was seen in the Raven's Progressive Matrices (RPM) sessions for both schools. This is not surprising, given that the RPM is a visual test. When asked about the way the participants solved the RPM problems, their responses were largely the same between the two groups. The following exchange involving Hannah and I is a good example of what the participants did:

Hannah: Yeah, oh I kind of get that [pause] yeah, yeah, I see. It's like saying what was subtracted from these two, like, taken away from the pattern, I guess.

Interviewer: Oh yeah.

Hannah: You could say [that], oh, when you look at it now anyway. [Extended pause]. [Excitedly] Oh yeah, I see that now! 'Cause it's that [answer], 'cause you've taken away the cross out of there, so that's what you've taken – the cross and the box – so that's what you've got [left] now [that] that's you've taken away; taken [the cross and the box] out of it. I see that now!

In the above exchange, Hannah excitedly reveals her solution strategy. First we see Hannah's use of mathematical ideas such as addition and subtraction to represent what was done to one figure to arrive at another. Next, she identified that a pattern was present within the question. Finally, she spoke about the cross and the box, which were constituent parts of the objects in the RPM question E5. She recognised the similarities and differences between where these constituent parts were present. Based on the pattern this produced, she was able to work out the answer. This insight excitedly took place, in its entirety, after an "Oh yeah!" moment – a moment of cognitive clarity.

4.4 The "Oh yeah!" Phenomenon

The "Oh yeah!" phenomenon that arose in this exchange occurred regularly throughout the whole range of problem solving sessions. The participants would be trying to work out a question with little success, usually silently, and then all of a sudden they would exclaim "Oh yeah!", or "I see that now!", or some sort of variation. After that, a period of cognitive clarity would follow where the participant would often be able to explain the rest of the path required to get to the answer. This moment of insight always occurred during a period of high cognitive load, after a period of cognitive confusion. This also often followed a period of scaffolding and then independent thought. The "Oh yeah!" moment of insight was associated with rapid cognitive load reduction, where the participants were able to fuse many formerly disparate sources of information into a single cohesive unit of understanding in working memory. Based on the knowledge of our working memory limitations and the reactions of the participants when they had sudden cognitive clarity, it is possible to put forward a tentative explanation for what might be occurring.

There may be a threshold dividing cognitive overload and cognitive clarity. This threshold is related to the maximum amount of information held in working memory at any one time. This threshold, according to Miller (1956) resides somewhere between 5 to 9 items, depending on the person. For oversight of a problem, and the understanding that comes with it, to occur, cognitive load must be below this threshold. Cognitive load reduction is caused by combining disparate elements into a single cohesive chunk. Certain reductions of cognitive load can be small (e.g. chunking 2 elements into 1) or some can be large (e.g. finding the relationship between 5 seemingly unrelated elements by identifying a pattern). Large reductions can occur independently or after scaffolding, because a framework in which to fit all the elements has been figured out or been given to the participant. Once the framework is in place, relationships "jump out" due to the new way of looking at the problem. It is these sudden, large reductions in cognitive load that lead to the "Oh yeah!" moment. Thus, that moment is merely one where a rapid reduction of cognitive load takes the overall cognitive load below the threshold required to achieve clarity. Once that clarity is achieved, a correct solution generally follows. Hannah and another participant, Jake, give a clear explanation of this.

Hannah: Sometimes in the test if it clicks, you don't, I dunno, you have all this information going through your head and then something will pop out or click to you or something and, yeah, you've just kind of got your answer. You go with what it tells you.

Interviewer: What about for you Jake, when it just clicks?

Jake: Well normally, I feel really happy, like "yay!" [excited]. Normally it's a step that I do and then it clicks. Like I might do a certain step that opens up everything else and then it goes "Bam!" (Interview Session).

There is evidence that when a person experiences a cognitive breakthrough or sudden success in interpreting information, the brain releases endorphins which act as a reward circuit (Biederman & Vessel, 2006; Greive & de Berg, 2008). Thus, the participants are likely to get excited and feel good about the "Oh yeah!" moment, as is suggested by Jake in the above exchange. Upon achieving a solution, the reward structures in the brain supply the participant with a positive feeling of motivation, confidence and the sense of a power over the ideas contained in the question – the solver can now use that knowledge to his or her advantage (Greive & de Berg, 2008). The physiological response of the body in releasing endorphins at the point of sudden cognitive clarity further reinforces the significance of the "Oh yeah!" moment to problem solving.

However, the "Oh yeah!" moment did not always lead to a complete and successful solution. In these cases, the original realisation was incorrect, there was a flaw in the participant's reasoning or there was still some data in the question that remained unaccounted for. At other times, an "Oh yeah!" moment actually occurred when the participant realised that what they had been doing was incorrect. There was sufficient clarity to tell the participant the path they were going down was wrong, but there wasn't enough to tell them what the right path was. In both of these cases, the "Oh yeah!" moment of insight still contributed to an eventual solution as it caused the participant to re-evaluate their position in light of their new insight. The "Oh yeah!" moment arose often during the sessions – on average, around 3 times in a 45-minute session. The number of times that the "Oh yeah!" moment occurred made it a phenomenon that needed to be addressed in the results chapter. While a whole research project could be undertaken on solely explaining this phenomenon, the interpretation given to it in the current study fits the data and is a good starting point for any future inquiry.

4.5 Mechanisms Governing Problem Solving Strategies

The preceding information suggests that the realisation of a solution strategy has generally been related to a restructure and a reduction of cognitive load; hence, cognitive confusion is resolved and the identification of problem type is facilitated. Having considered the role of reading, internalising, and classifying of problems, and the participants' use of solution strategies for managing information and restructuring problems, a need arises to explain the mechanism by which these are governed. Such a mechanism does exist, but it appears as though it consists of a combination of two factors that govern how a participant will use the reading and classification of problems and solution strategies. The factors that mediate how problem solving techniques are used are expertise and metacognition.

4.5.1 Expertise and prior knowledge

The effects of expertise are well documented in the literature of problem solving (see de Groot, 1966; Chase & Simon, 1973; Sweller, 1999; Sweller, van Merriënboer & Paas, 1998). While this study did not contain experts in the traditional sense of the word, there was strong evidence suggesting that some participants displayed a high degree of competence in certain problems that were of a very familiar kind to them. Performance of this kind was present in problems where the participants had a high degree of prior knowledge (see questions 1 and 2 in Appendices A and B). In these questions, prior knowledge allowed the participants to instantly identify problems and traverse the correct solution path without hesitation, deftly reaching a correct solution with absolute cognitive clarity throughout the solution process.

We know the novice-expert dimension is a continuum, not a dichotomy. With that in mind, we would expect a continuum of problem solving proficiency that is dependent on the degree of

expertise. For novice problem solvers, there is no extensive background, therefore there is very little guidance coming from schemas in long-term memory. The only approaches available to them are inferior approaches such as means-ends analysis or trial and error. However, as expertise increases, so too does the solver's ability to use schema-based problem solving approaches. In the intermediate levels of expertise, these schemas may be available only for a section of the problem. All other sections require inferior problem solving approaches. If there are gaps in the participant's knowledge at critical parts of the question, a solution may be impossible to find even though the participant can solve all of the remaining constituent parts of the problem. It is at these points that scaffolding could provide a significant benefit to the problem solver. Expert problem solvers will have a previously constructed schema in place which will guide their problem solving from beginning to end. These people will have no trouble with the question. Evidence for this continuum phenomenon was widespread throughout the present study, where participants showed schema-based problem solving in parts of the question and means-ends analysis in other parts of the question.

4.5.2 Metacognition

Among other functions, metacognition can be viewed as the process by which cognitive load is monitored. In this study, metacognition was portrayed to the participants as the internal voice that spoke to them, guiding them through the problem by telling them what to do and how to do it. Metacognition allows a person to adjust their solution strategies in response to their cognitive load. The metacognitive process can also gauge the effectiveness of strategies as evidenced by their effect on cognitive load and distance between the current state and the goal state. It can then tell the person whether to persist with their current strategy or to try a new one. Thomas gave a detailed description of how metacognition helped him in his problem solving.

Interviewer: Do you ever monitor your own thinking when you're solving a problem ... think about what you're thinking and you kind of walk yourself through the problem?

Thomas: Yeah, probably. Yeah, ummm, yeah I think I do that sometimes, like with the more difficult questions. I probably say, "Hey Thomas" to myself, "umm, I think you can do this. Just work through this step by step and imagine what you're gonna do and then think about it [pauses] and think about, umm, think back to past references when a question like this has come up" and then just try and do it as best as I can really.

Interviewer: And how does that help you?

Thomas: What, thinking back?

Interviewer: Yeah, like organising your thoughts and telling yourself what to do and how to do it.

Thomas: I reckon it probably, ummm, calms me down a little bit, like say if I'm really stressing out 'cause it looks really hard. I probably just go back and it calms me down and then I think, "well, this isn't gonna be really hard, just settle down and think about it."

Interviewer: So if you were doing that ... talking to yourself sort of thing, what would that sound like?

Thomas: Oh, probably just saying to myself, umm [pauses], "think back Thomas, ... it doesn't look too hard, just look at it and then [pauses] really figure it out, think about what teachers have said to you that you can do and just settle down and really read through the question." That's probably what I'd say.

Interviewer: Oh yeah. And, do you think that with that monitoring [of] your own thinking, do you tend to do it more for the easier, or middle of harder questions?

Thomas: Probably for harder questions.

Interviewer: Yeah. Do you feel you need to [use metacognition] for the easier ones?

Thomas: Not really. Nah, not at all.

In this transcript, five features of Thomas' metacognition can be identified. First, it is only present during the difficult questions. This is also the period where cognitive load is expected to be highest. Second, there is an emotional aspect to metacognition. Thomas uses his metacognitive voice to calm himself down. This allows reasoning to take over and filters out any emotions of anxiety that may be affecting his cognitive capacity. Third, Thomas' metacognition encourages a systematic approach to problem solving. Working through a problem step-by-step allows him to limit the problem space and therefore the cognitive load imposed on him. Fourth, his metacognition forces him to reach into long-term memory for any relevant schemas. This process may facilitate the recollection of any schemas that might have initially escaped him. Finally, the metacognitive voice gives Thomas confidence.

Wendy and Hannah also comment on metacognition; however, their comments are somewhat different.

Wendy: Well if [the question is] really easy, I don't have a voice inside my head.

Hannah: Yeah, if [the question is] really easy, like 1+2, you think "oh duh!"

Wendy: But if [the question is], like, medium, I don't know. It's kinda like [the metacognitive voice is] there for part of the question...

Hannah: For the harder parts.

Wendy: Hmm (agrees). And [for] hard [questions], it's, like, there telling me to give up.

Wendy and Hannah confirm Thomas' assertion that metacognition is present for the harder questions or parts of questions. One important difference is the emotional effect that metacognition had on the two participants. While Thomas' metacognitive voice settles his down and spurs him on, Wendy's metacognitive voice tells her "to give up". Thomas and Wendy have very different personalities; Thomas is confident in his ability whereas Wendy is not confident. The metacognitive voice then may reflect certain qualities of a person's personality. If a participant has a certain level of self-efficacy and confidence, one would expect the metacognitive voice to have the same qualities, or at least convey messages to that effect.

Metacognition appears to have a two-pronged effect. First, it mediates the problem solving process by serving as a voice that guides the learner in choosing what strategies to use and when to use them. Second, it acts as an emotional feedback mechanism which can either urge the participant to persevere or to give up. However, emotional responses to cognitive overload are only one of the many personal factors that can affect the way participants traverse problem solving paths.

4.6 Personal Factors[3]

A number of factors that are intrinsic to the problem solver were observed. The first three that will be discussed are confidence, self-efficacy and perseverance. While their effect on metacognition has been discussed, there was also evidence for a direct effect on error rates and ease of solution.

4.6.1 Confidence, self-efficacy and perseverance

Wendy is a participant with low problem solving self-efficacy and confidence. Comments that Wendy made to that effect include:

- I think it's just me finding [the Raven's test] hard!
- But, like, [the Raven's problems] are probably really obvious to people like Jake and Hannah. I just stare at them.
- I don't think I did... did I do this right? It's so embarrassing!
- I feel so dumb!
- Well I don't get it, this is really sad!

It's therefore no surprise that her metacognitive dialogue is also negative. This negative thinking seemed to interfere with her cognitive clarity. She was so worried about underperforming or appearing dumb that she had trouble focussing on her work. Wendy also had a tendency to compare herself with the other participants which did not appear to directly

[3] Prior knowledge and expertise is a major factor which fits under this category, but as it has already been considered earlier in this chapter, I will include it in personal factors without further analysis. The key point to be made is that expertise is a unique personal factor as it is also one of the mechanisms governing solution strategy use. Above all else, expertise shapes the solver's perception of a problem, and thereby its solution.

impact her problem solving as much as her low self-efficacy and confidence did. However, this comparison was yet another concern in her mind competing for scarce cognitive resources.

Having a low self-efficacy causes the participant to think they cannot successfully solve a problem. Believing they cannot do it then calls into question the need to persevere. If they believe they will never arrive at a solution, then perseverance is futile. This results in learned helplessness. However, for participants such as Thomas, Matthew or Jake, who had high self-efficacy and confidence, perseverance came naturally to them. This was particularly true for Jake, who displayed a genuine desire to successfully solve every question. Jake experienced a sense of joy and empowerment coming from discovering a correct solution. This sense of joy and empowerment appears very similar to the physiological response that is present in the "Oh yeah!" moment. The desire for this physiological and psychological reward drives the participant to want to solve more problems and persevere with the current problem. This extended dialogue with Thomas explains it well.

Interviewer: How does it feel for you when you are able to solve a problem and get it right?

Thomas: I probably get more confidence for the next question. I think to myself, "Well, if you can do this, you could probably do anything harder in this test, when these questions come along."

Interviewer: And does that confidence help you later on?

Thomas: Yeah, I reckon it does.

Interviewer: Yeah, so when you're not confident, how do you feel about, like, trying to answer questions?

Thomas: Well, if I really don't go into it confident and I'm really nervous, I don't think I'll go well.

Interviewer: Yeah.

Thomas: So really, if I get to a hard question and I'm really not concentrated, I normally skip it and go to the easiest questions and get into the state of mind where I'm thinking that I can really do this and that I've got the brain power; I've learnt this stuff and I know that I can do it. And then I go back to the hard questions.

Interviewer: So you actually, like, do the questions in the order that you reckon will build your confidence the most?

Thomas: Yeah, I reckon I do. (Interview Session)

For Thomas, confidence is an important factor in problem solving. Thomas goes out of his way to bolster his own confidence and this prepares him for more difficult questions. We then moved onto discussing his motivation for problem solving.

Interviewer: You said that when you solve a question, you get more confidence, right?

Thomas: Yeah.

Interviewer: And you kind of feel like you've got power over that question, like you know how to solve it now. Yeah?

Thomas: Yeah I reckon that, like, I feel that I'm kind of like, on top of the world. (Hesitates) Aaah, not really like that but [pause] like I've got the feeling that, of, umm (searching for the right word) belief; kind of.

Interviewer: And that you've kind of mastered that particular thing, right?

Thomas: Yeah.

Interviewer: Do you think that that mastery over the question or over the knowledge motivates you to try harder and to do well?

Thomas: Ummm, yeah, I think it does.

Interviewer: I guess...

Thomas: (Interrupts) I reckon it gives me a boost to go, ummm, say if I'm really distracted, I reckon if I do a question like that, like a hard one, I'll be like "oh yeah, I don't reckon this [hard question] will be [hard] anymore if I can do this [hard question]."

Interviewer: So does that kind of drive you to want to solve more problems?

Thomas: Yep. (Interview Session)

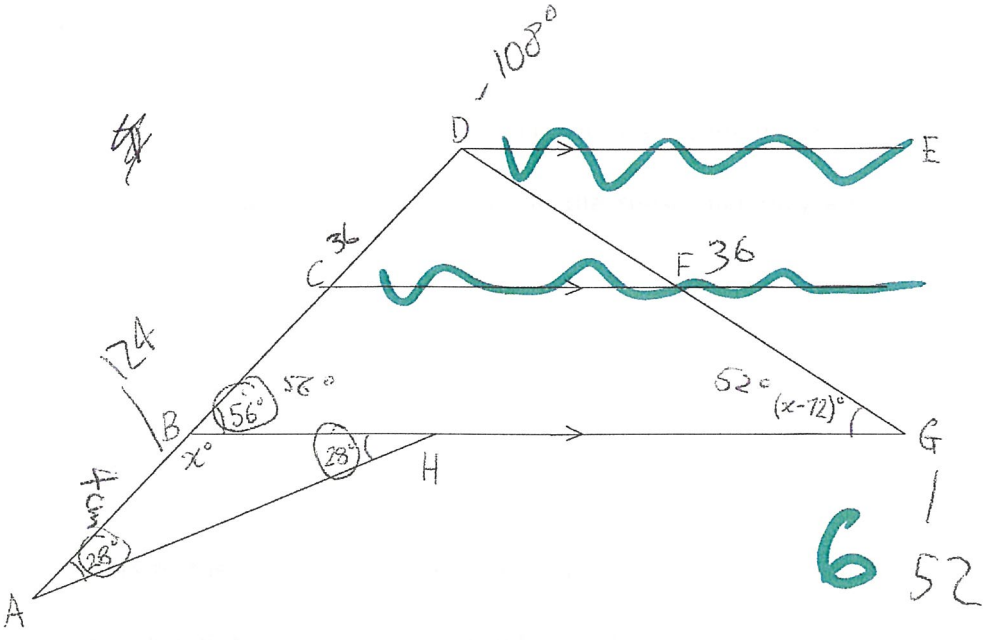
For Thomas, being able to solve a question successfully is a great motivator to want to continue to solve questions like it. Further, success in problem solving feeds back to Thomas' self-efficacy and confidence in a positive way, increasing his confidence, self-efficacy and motivation; or as Thomas himself puts it, "get into the state of mind where I'm thinking that I can really do this and that I've got the brain power."

4.6.2 Cognitive constants

Along with the three psychological personal influences described above, there are also other personal factors that directly constrain or demarcate the way an individual's cognitive system functions. The collected data suggests that fluid intelligence[4], problem solving style, learning style and creativity in problem solving all have an effect on the way a participant will approach

[4] Fluid intelligence represents an individual's generalised ability to think clearly and deftly. The RPM test used in this study is a highly regarded test of fluid intelligence.

a problem. To illustrate these effects, Rosie, a creative problem solver with a strong visual learning style, approached Question 9 of Session 1 by mentally filtering out two lines that she considered irrelevant to the problem. These two lines are crossed out in figure 4.3. About those two lines, Rosie stated that, "I just pretended they weren't there ... in my head, I got rid of [line CF] and [line DE]." (Geometry Session). These lines were filtered out to such an extent that she "didn't put [the lines] back [into her mind]." This creative way of reducing cognitive load came so naturally to her that initially she didn't realise that she had done it. In contrast, more algebraically oriented participants such as Thomas or Hannah just solved the problem formally, as they would do for any other angle-finding problem. While they were effective in their problem solving, they were nowhere near as creative as Rosie. Their progress in solving the question may have been similar, but their approaches were very different.



9. Find the value of as many angles and sides as you can, giving reasons for each. You may calculate the angles and sides in any order.

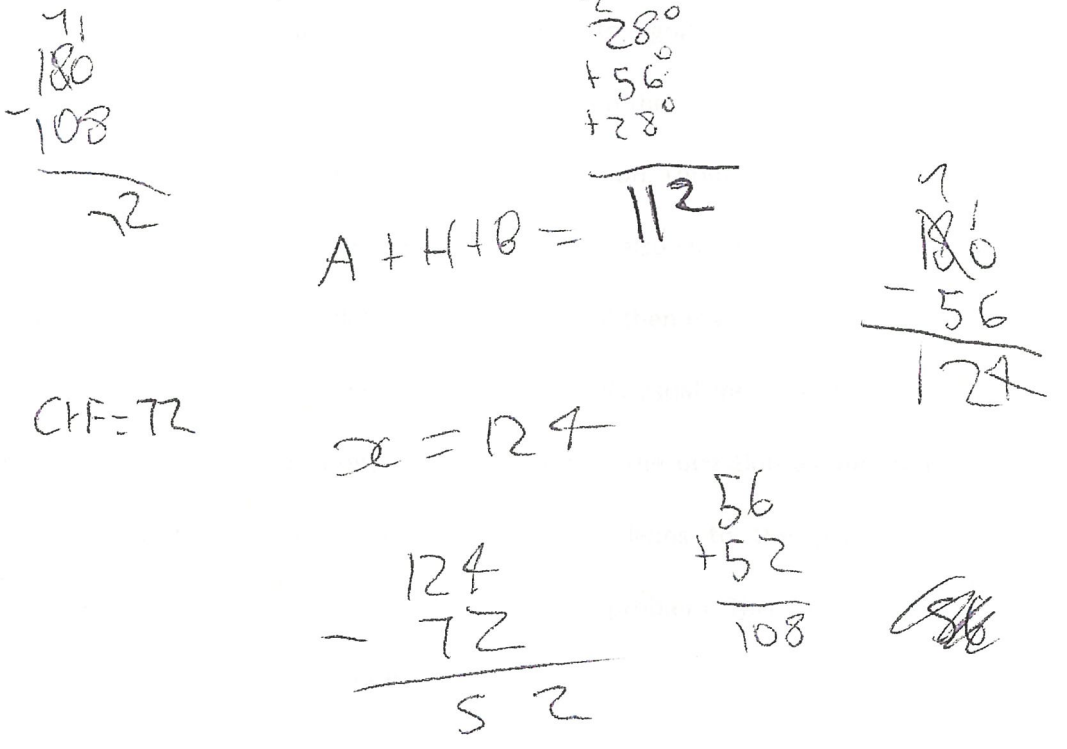


Figure 4.3: Rosie – Session 1 (Geometry), Question 9. All writing is Rosie's except for the two green lines covering lines CF and DE, which I drew after the question was solved to illustrate Rosie's mental filtering. The "6" is Rosie's subjective cognitive load rating.

4.7 External Factors

There are also a number of factors that affect the learner, yet they are not directly controllable by the learner. These are known as external factors in the sense that they are externally imposed on the learner.

4.7.1 Task significance

Task significance refers to the value that the participant puts on a high performance in the task. The task significance also contains a component of intrinsic significance, e.g. a yearly exam has a much greater intrinsic significance than a small class test. Now, problems that occur during regular class times may be given very low values of significance, but when an identical question appears in a yearly exam, its significance greatly increases. The data suggests that task significance has a direct effect on the care and attention to detail taken in a problem solving process. In the final interview session, Thomas suggests that an easy question "doesn't really make me think." However, Thomas also states, "Say, if [the question] is in my final exam, I'll probably check through it... [I'd also] then read it again, and then, to make sure 'cause it's a finals exam". This is in contrast to his usual method of solving easy problems, which is to "get [the question] sorted out and do the question as quickly as possible." The greater importance placed on significant tasks leads to the participant putting more perseverance into and taking more care with the problem. This has an effect on error rates, and therefore, successful solution.

4.7.2 Scaffolding and assistance

Scaffolding by an expert can help transform cognitive confusion into cognitive clarity by the means of well-timed acts of guidance. It is possible that a participant can develop 95% of the steps in the question but cannot resolve one critical component of it. This leads to a breakdown in the problem solving process, even though the participant knew almost all about how to solve the problem. A reason why the "Oh yeah!" moment can occur after a scaffold is processed is because the assistance given may have contained or facilitated the discovery of the single piece of information that was preventing the participant from accessing many parts of the question. Once the single piece is processed and related to the rest of the problem, clarity rapidly follows, assuming the possession of schema-based knowledge for the rest of the question elements. However, scaffolding requires a certain amount of skill on the part of the scaffolder to phrase the assistance appropriately and to pick the right moment in which to provide it. Scaffolding can be thought of in two ways: helping participants with problem processes and helping them with problem knowledge.

4.7.2.1 Scaffolding – Problem Processes

An example of scaffolding comes from the interaction with Thomas, who was struggling with question 4 of the geometry paper (see figure 4.4).

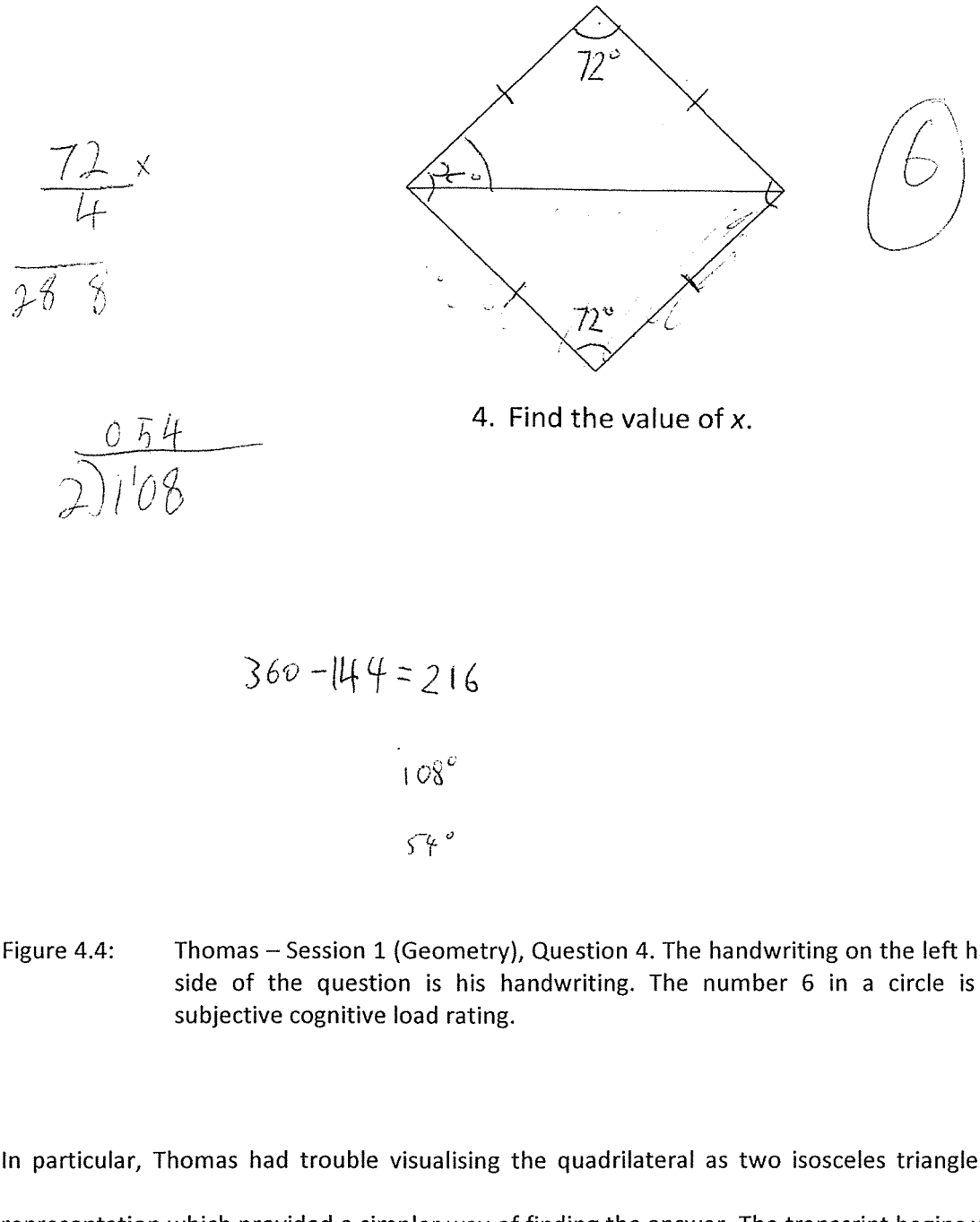


Figure 4.4: Thomas – Session 1 (Geometry), Question 4. The handwriting on the left hand side of the question is his handwriting. The number 6 in a circle is his subjective cognitive load rating.

In particular, Thomas had trouble visualising the quadrilateral as two isosceles triangles, a representation which provided a simpler way of finding the answer. The transcript begins with Thomas having spent 3 minutes on trying to figure out the question using the angle sum of a quadrilateral. That particular piece of working is evidenced by the 72 x 4 = 288 on the left hand side of figure 4.4.

Thomas: (Hesitantly) So if that's all 280 degrees... 288... that doesn't make any sense. A bit confused. Ummm...

Interviewer: A bit confused? All right, well let me help you out. What if I got rid of all that bit at the bottom? [I then crossed out the bottom triangle]

Thomas: Just made it a triangle?

Interviewer: Would that help you?

Thomas: So just trying to find... so that the triangle will be 180 then.

Interviewer: Yeah, would that help you?

Thomas: So you'd take 72 from 180, which would be 102 I think. 108. So 108 divided by 2 would be (works 108 ÷ 2 out on the paper)... so it'd be 54 degrees.

The fact that the above exchange took place in only 45 seconds, measured from the time that I began to provide the scaffolding, displays the high level of clarity in Thomas' mind after I gave him guidance on the problem process. It can be noted that Thomas never actually answered the question, "Did that help you?" I believe this was because his mind was so focussed on the new clarity that he did not feel the need to address the question directly; all his cognitive resources were focused on the finding the solution.

The discussion after the question was completed provides some more insight into how the scaffolding helped to reduce cognitive load.

Interviewer: You seemed a little bit confused until I rubbed off the bottom half of the...

Thomas: Yeah.

Interviewer: ...thing and then you saw that it was just a triangle. Do you think that having that extra stuff on the bottom actually made it harder for you to solve the problem?

Thomas: I reckon... Yeah, I reckon that it actually made it, umm, maybe twice as hard 'cause you've got twice as much to figure out.

Thomas went on to say that if he was given extra information that he didn't actually need to solve the problem, this extra information made the problem harder to solve. From a cognitive load perspective, it can be seen that when Thomas states, "you've got twice as much to figure out", he is referring to the fact that there is twice as much information to process before determining a solution path, as compared to a question with only one isosceles triangle. There is actually still the same amount of information to *figure out* in both questions; namely, the value of *x*. This comment by Thomas suggests that redundant information makes it more difficult to find a solution – a conclusion established by cognitive load researchers for over 10 years (see Jamet & Le Bohec, 2007; Sweller, 1999). The fact that redundant information increases cognitive load due to the need to process extra information that does not inform the problem solving process has been termed the redundancy effect (Sweller, 1999). The scaffold provided in this instance explicitly pointed out to Thomas that the redundant information was not necessary for a solution, and thus caused Thomas to discard the redundant information which consequently reduced his cognitive load. This reduction of cognitive load led to instant cognitive clarity and a rapid and correct solution.

As an aside, an expert problem solver in this area would realise the presence of the two triangles and either mentally filter out the redundant information or use the angle sum of a quadrilateral to solve the problem. Thus, the redundant information would not provide a significant roadblock for a true expert. They would therefore not need any scaffolding, even though the same redundant information is present for both problem solvers. This further

highlights the great significance of the role that prior knowledge and expertise plays in the problem solving process.

4.7.2.2 Scaffolding – Problem Knowledge

While solving question 7 of the geometry section (see figure 4.5), Matthew had some trouble with the amount of information present in the question.

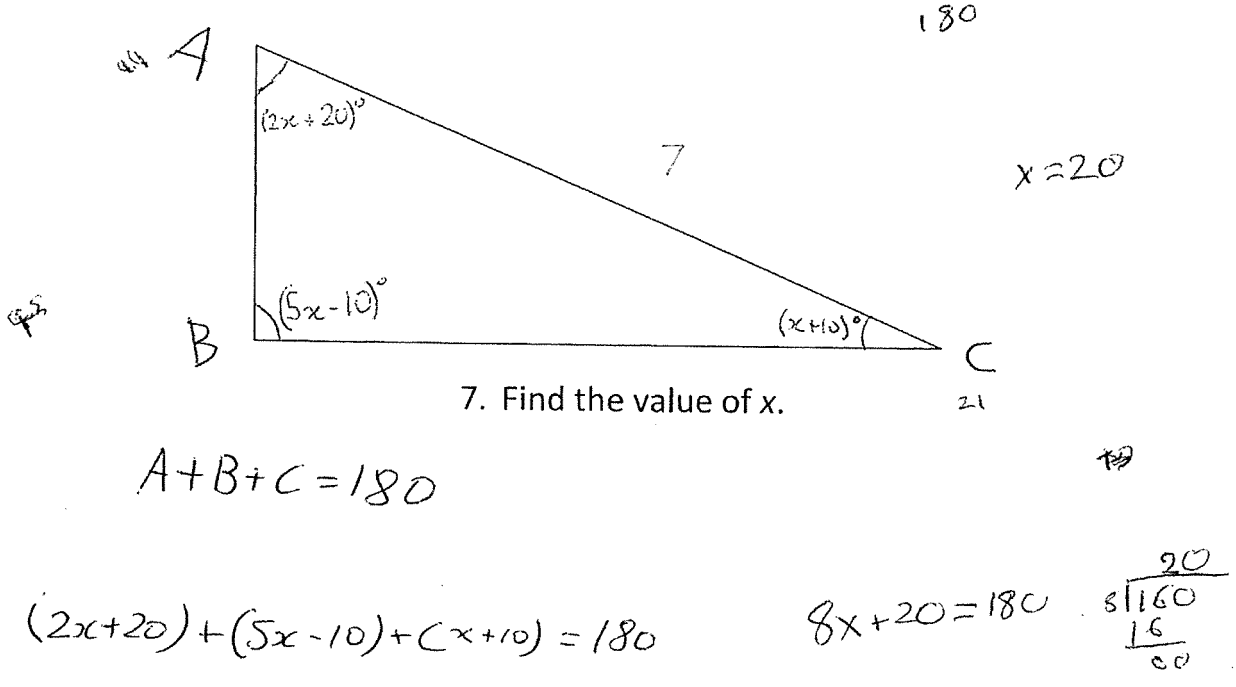


Figure 4.5: Matthew – Session 1 (Geometry), Question 7. The blue handwriting is mine. The green "7" above the hypotenuse is Matthew's subjective cognitive load rating.

Interviewer: Do you find that there's a lot to think about in this question? Like a lot of information?

Matthew: Yeah.

Interviewer: So how are you going to deal with all the information that's there?

Matthew: So first you think of a number that would be suitable for *x*. And then you work out all of them with using that number as *x*. And then if they all plus up to 180, then you're right.

Interviewer: So in other words, you're using trial and error.

Matthew: Yep.

Interviewer: Can you think of another way that you might be able to do it?

Matthew: Umm, my teacher's told me another way but I... just talked... through it. (Laughs)

Interviewer: (Laughs) Well thanks for the honest answer at least! I'll give you a clue: it does have something to do with your beloved algebra.

Matthew: Oh, no way, I hate it.

Matthew knew it was an angle sum of a triangle question and had realised that all three of the angles added to 180 degrees. However, he did not make the connection in his mind that this could be turned into an equation and solved using algebra, and so dropped back to using trial-and-error. This is a very inefficient method when luck is not on the side of the solver, and its success rate decreases as the complexity of the question increases. I gave him the hint to use algebra without any mention of how it might be used. At this stage, the only scaffolding provided was related to problem processes.

After this exchange, there was a pause for about 10 seconds, after which the conversation resumed.

Interviewer: And let me know if you want another clue.

Matthew: Nah, I'm not into the whole algebra thing, so probably not.

Interviewer: It's the only way you'll be able to solve it in a decent amount of time. 'cause otherwise you'll just have to guess values for x and it will take you all day. (Pause) So if this angle's A, and that angle's B and that angle's C (writing the angles into the question in blue pen – figure 4.5), can you write, like, an equation that tells me what they all add up to? (Pause) Like what are, what was A + B + C?

Matthew: A + B + C = 180.

Interviewer: 180. Good. So that's... and I'll write that down for you 'cause that's a pretty big clue.

Matthew: (Pause) I still say my way's better. I can think of a couple of, ah, 3 [values] that would actually equal up to it but yeah.

Although I gave Matthew the main step required for a successful solution of this problem, he still insisted on using trial-and-error rather than algebra. He tried to substitute $x = 10$ but when he saw that only added up to 100 degrees, he thought "well, I've got 80 left" so he tried to use the 80 degrees information to somehow find a better approximation for x. The next value that Matthew tried was $x = 11$. This didn't work so he guessed $x = 18$. After spending nearly 2 minutes guessing, I decided to show him the proper solution process. I was not intending to scaffold; at this point, I just wanted to show him how to get the answer. Matthew, however, had other ideas.

Interviewer: 18's really close. I'll show you the way that I'd solve this. So you know that A + B + C are 180, right? So what I do is in place of A, I write the value of A. So that's $2x + 20$. And then I add it to B, which is $5x - 10$, and then I add it to C, which is $x + 10$. And all that...

Matthew: Oh!

Interviewer: ...adds up to 180.

Matthew: I get you now.

Interviewer: Do you get it?

Matthew: Yeah, so it'd be... $8x$... (Working out on paper) ... plus 20. (Tentatively) So you'd go 180 [pause] divided by 8.

Interviewer: Well $8x$ plus 20 equals what?

Matthew: 180.

Interviewer: So you might want to write that down.

Matthew: Oh (Realisation that something is not quite right).

Interviewer: 'Cause [having numeral and pronumerals together will] change things again.

Matthew: And then... Oh! So you'd minus 20 from 180 first, so that'd be 160. And then you'd want 160 divided by 8... 8. (Working out on paper) 20.

Interviewer: So that gives you what?

Matthew: It'd be 20. $x = 20$.

Interviewer: Good.

Matthew was able to come up with a solution after being scaffolded with the correct problem knowledge. Filling in the gaps in Matthew's knowledge allowed him to find a solution that he otherwise could have not found through mathematically sound methods. It was clear in this case that Matthew's problem solving process was fine; however, he was just missing some key elements of knowledge that would have allowed him to solve the question. Giving him the correct information to use at key points in the question allowed him to reduce his cognitive load by knowing what piece of information to use at what time. He could then devote all of his remaining cognitive capabilities to figuring out how to use that piece of information. Overall, then, it can be seen that scaffolding allows gaps in prior knowledge or solution strategies to be filled, which allows a redistribution of cognitive capabilities to the areas where they are most demanded by the problem being solved. We also see that scaffolding allowed "Oh yeah!" moments to arise. There were two in the section of transcript just quoted and both led to significant cognitive clarity and breakthroughs in the problem. The scaffolding process allows problems to be solved which would otherwise prove unable to solve.

4.7.3 Distractors

There are often external distractions which also impact on problem solving capacity. For instance, external noise, friends walking by, hunger and tiredness all contributed to a loss of concentration or temporary breakdown in the problem solving process. These factors have the potential to create thoughts that can interfere with working memory processes.

4.8 Discussion

The factors discussed in the results chapter have all been shown to have an effect on problem solving. A model synthesised from my knowledge of the literature and reflection upon these

results has been proposed which graphically shows the part each of the above factors plays in the problem solving process (see figure 4.6). The next two sub-sections of the discussion describe the model in detail.

4.8.1 The Appraisal Stage

As can be seen in figure 4.6, there are two stages to problem solving. In the appraisal stage, the student initially reads, internalises, identifies an initial problem state and classifies a problem into a certain category. The significance of the task and various personal factors determine the way in which the reading, internalisation and classification process takes place. Then, based on the degree of expertise of the learner, appropriate forms of mental manipulation and optimal solution strategies are initially decided on. These feed into a cognitive load that arises from the initial processing of question. Of course, the cognitive load is present from the very beginning of the reading, internalisation and classification stage, but for the purposes of this diagram, it is represented as a single entity. It can also be seen that during the appraisal stage, all the effects are one-way. This is due to the appraisal stage being an initial stage of problem conceptualisation and encoding only – no solution-related processing occurs in this stage.

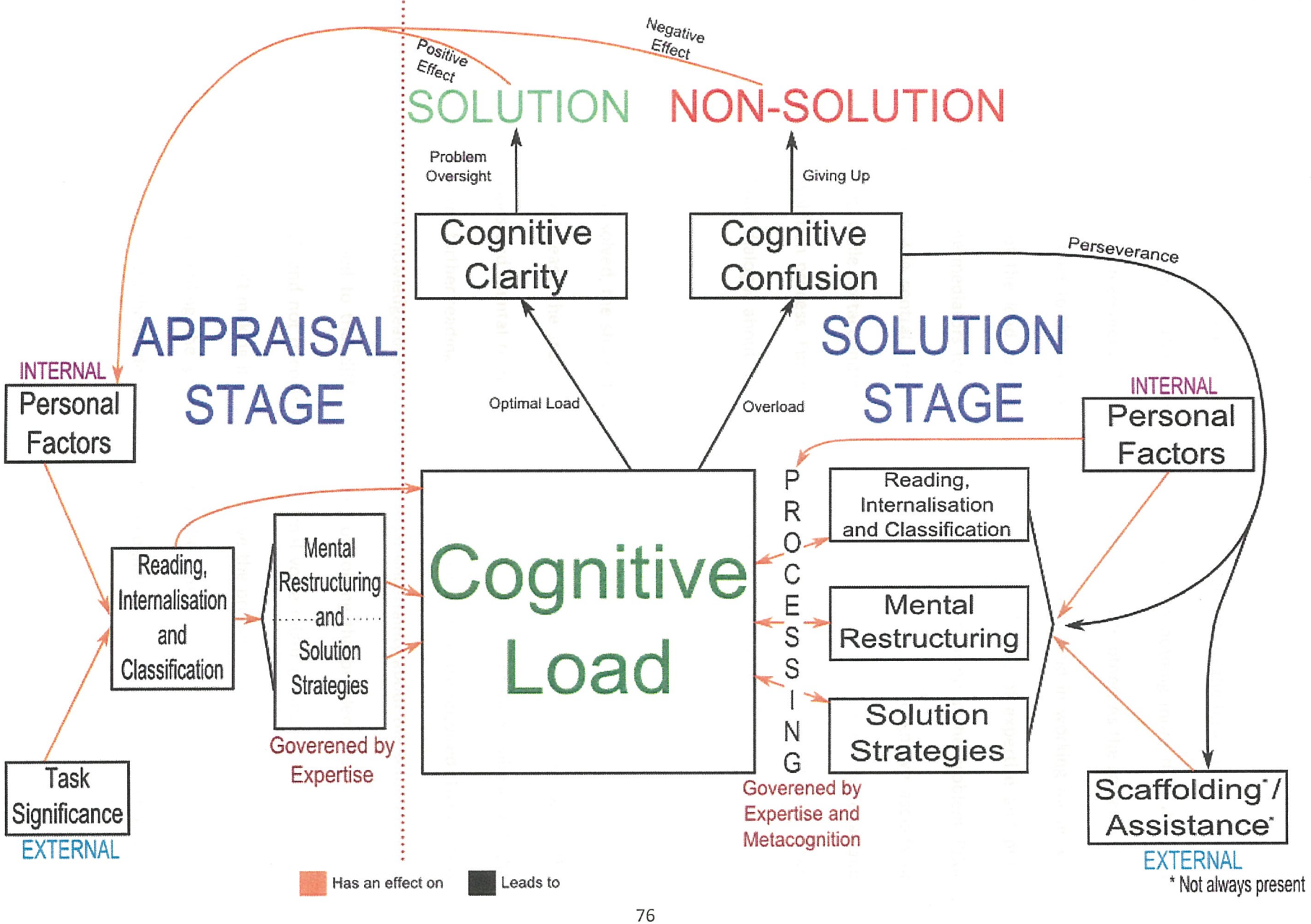


Figure 4.6: The proposed model for cognitive load-centric problem solving.

4.8.2 The Solution Stage

The cognitive load that is carried over from the appraisal stage forms the baseline from which the solution stage proceeds. The cognitive load, of course, is nothing more than the load imposed by the question elements and their relationships to each other. As the solver passes from the appraisal stage to the solution stage, the information held in working memory is analysed in terms of the load present. Metacognition and degree of expertise and prior knowledge act as the mediators for the rest of the processing involved in the problem. Prior knowledge informs the central executive governing the mathematical cognitive aspects of problem solving, while metacognition handles the confidence, self-efficacy, emotional and monitors the solution process. Further, metacognition also acts as the mechanism by which mentally "audible" dialogue about cognitive load and the required strategies comes about.

As the problem is solved, the student will attempt different ways of solving it. If they find the problem to be very easy, the student will have a path ready to follow. They will know the solution strategies and mental restructuring required to solve the problem, along with the times at which any further reading, internalisation and classification are required. The process will be fluent for a knowledgeable student. However, if uncertainty about the solution exists, then students will need to trial different methods of solving the problem. For instance, if cognitive load is high and not decreasing, the internal voice might produce something like, "What you are trying isn't making it any easier to solve the problem. Maybe you should try a different strategy/re-reading the problem/putting the problem in a different form." This process will continue until the student reaches either cognitive clarity or cognitive confusion.

A reduction of cognitive load below the threshold that causes overload is necessary in order that the student gains cognitive clarity. If this cognitive clarity is present at the end of the problem solving process, the productive performance of the student will lead to a correct solution. However, if the student is in a state of cognitive overload, they will inevitably move towards cognitive confusion. At the state of cognitive confusion, there is an important decision to be made. If the student decides to persevere, they can re-enter the problem solving process and continue in their attempt to achieve cognitive clarity and a correct solution by attempting a new approach through the restructure of the problem state. However, at this point they can also decide to give up. This will lead to a non-solution. There is no provision for an incorrect solution because this model assumes the student is getting instant feedback about whether their solution is correct or not. In a situation where they don't know if they are correct or not, a situation could arise where they believe they have achieved cognitive clarity and a correct solution when in fact, they haven't. In this case, upon learning their solution is incorrect, the student would then have the choice to persevere towards finding a correct solution, or give up and leave the problem unsolved. Thus, this special case is also explained by the proposed model.

The two other aspects feeding into the solution stage of problem solving are personal factors and scaffolding/assistance. Personal factors not only have an effect on the three ways in which the problem is solved, but also a direct effect on processing as mediated by expertise and metacognition. Some evidence of this is provided by the way that metacognition changes due to varying levels of self-efficacy and confidence. This changes the metacognitive dialogue in the student's mind and affects their choice of problem solving strategies.

Scaffolding and assistance are key aspects of the solution stage. After a student has gone through cognitive confusion and has decided to persevere with the problem, they will continue to languish in cognitive confusion unless a new insight is achieved. Often, this insight can be achieved through the student's own problem solving processes, but sometimes it cannot be achieved. Scaffolding and assistance provide an external means by which an insight can be achieved.

Finally, irrespective of whether a solution is achieved or not-achieved, there will be an impact on the learner's personal factors, particularly confidence and self-efficacy. A successful solution will positively affect confidence and self-efficacy, while a non-solution will negatively affect them. For instance, Thomas states, "[If I get a question right], I probably get more confidence for the next question. I think to myself, 'Well, if you can do this [question], you can probably do anything harder in this test'."(Interview Session). Further, a solution will reinforce prior knowledge and strengthen the particular method that was used to solve the question. A non-solution can inform the student about correct problem classification and unsuccessful problem solving techniques, but it cannot create a schema for a correct solution.

Chapter 5

Implications and Conclusion

5.1 Introduction

This study has explored the role of cognitive load theory in explaining the problem solving process. Participants' problem solving processes were studied through interview sessions, Raven's Progressive Matrices and geometry and algebra problems. During the first (Raven's) session, the six grade 7 or 8 participants solved the final 48 problems in the Raven's Progressive Matrices test. After completion, they discussed their problem solving methods. The second session (geometry) and third session (algebra) consisted of participants solving problems and verbalising their solution strategies. The final session consisted of an interview in which the participants were asked specific questions about problem difficulty-dependent solution strategies, cognitive overload, the "Oh yeah!" moment and metacognition, along with any other topics that may have arisen during the interview. All sessions were audio recorded and the relevant sections were transcribed and carefully analysed according to increasingly refined categories to obtain the results of this study.

5.2 Research Questions

This study has proposed four research questions. These are listed below:

1. Can cognitive load theory be used to explain observations of student problem solving performance?

2. If so, can a model incorporating cognitive load theory be created to guide teaching for problem solving?

3. What implications does the model hold for teachers?

4. What implications does the model hold for students?

Section 5.3 addresses the first two research questions while section 5.4 addresses the final two.

5.3 The Construction and Application of a Cognitive Load-centric Model of Problem Solving

The analysis of the interview transcripts, participants' solved problem booklets and field notes gave the researcher a broad but detailed insight into the nature of cognitive load in problem solving. The cognitive load perspective from which this study was undertaken guided the researcher in interpreting the results. It was found that cognitive load is an influencing factor in the problem solving process in a variety of ways. First, the student reads a problem, forms an internal representation of it and classifies it by comparing the problem's characteristics to those of other known problem types. Personal factors such as confidence, self-efficacy and perseverance combine with external factors such as task significance to affect the way in which a problem is initially read, represented and classified. The participant then moves into an initial stage of processing where the problem is restructured (if necessary) and optimal solution strategies are decided upon. This stage is governed by the degree of expertise. This appraisal stage of problem solving occurs very rapidly. It is instantly after the first contact with a question during the appraisal stage that an initial cognitive load is formed. This cognitive load consists of the participant's initial impressions of the question, as imposed by the process taking place in the appraisal stage.

The cognitive load arising from the appraisal stage is the cognitive load that determines the progress of the solution stage. The participant's prior knowledge will determine the quality of the activated solution schemas. These schemas will govern whether the participant must re-read, re-internalise and re-classify the question, use a mental restructuring technique or use a particular solution strategy. Metacognition gives the participant feedback about the effects their chosen solution path is having on their cognitive load. This is a continuous process where metacognitive and problem progress information is constantly being fed back to the participants, guiding them in choosing a solution path and implementing its associated mathematical manipulations. The same personal factors that influence the appraisal stage will also influence the choice of solution methods in the solution stage.

If this process results in a manageable amount of load from beginning to end, the participant achieves cognitive clarity, which brings with it an oversight of the problem and a successful solution. However, if at any time, the student encounters more information than can be successfully processed in working memory, a state of cognitive overload will be produced which leads to cognitive confusion. If the participant chooses to give up, there will be no solution. If the participant decides to persevere, they can continue attempting to solve the problem. If no new breakthroughs are made, the participant will continue to flounder in cognitive confusion unless external scaffolding is introduced. Scaffolding introduces new information or restructures old information in a way that cognitive load is reduced, by building a structural framework that gives the participant a means by which to create understanding about a particular question aspect. Placing disparate problem elements into a structural framework facilitates chunking which leads to a reduced cognitive load. A reduced cognitive load breaks the cycle of cognitive overload and confusion and allows normal problem solving procedures to resume.

Finally, the successful solution of a problem will have a positive effect on personal factors such as self-efficacy and confidence, along with reinforcing all or parts of the chosen solution path. A non-solution will have the opposite effect on personal factors, but some parts of the solution path may be reinforced while others may be deprecated. The outcome of a problem solving process thus impacts any future related problem solving processes through this feedback loop.

The implications of this model for teachers and students will be discussed in the next section, matching the factors mentioned in the method with teaching strategies found to be effective by Hattie (2009). Of the methods mentioned in the model, self-efficacy, confidence, concentration, perseverance, scaffolding, cognitive load reducing aids, teaching for problem solving process, self-verbalisation and self-questioning, feedback and teacher clarity were found to have positive effects. A number of other teaching strategies related to this study's subject matter were linked to effective learning; however these were not directly addressed by the results.

The first two research questions, then, can be answered in the affirmative.

5.4 Implications for Teachers and Students

The results and the model that this study proposes recommend a number of strategies that could be employed by teachers to improve their students' problem solving performance. The two research questions considered in this section will be integrated into a single section

examining how teachers can teach for successful problem solving. This section will consider the practical implications of how this research could affect teachers and students.

5.4.1 How can teachers teach for successful problem solving?

There are many ways in which a teacher can teach for successful problem solving. These can be classified into cognitive approaches and psychological approaches. Hattie (2009) has authored a synthesis of over 800 meta-analyses which identifies the relative strength or weakness of various teaching techniques. Hattie's book uses the Cohen *d* statistic to indicate the link of a certain teaching technique with its ability to promote student learning. In Hattie's book, this *d* statistic serves as a numerical measure of each technique's effectiveness. Hattie (2009) argues that evidence of learning as a result of an intervention applied to an experimental group becomes, not just significant, but 'visible' when the *d* statistic exceeds 0.4.

5.4.1.1 Cognitive Approaches

In terms of cognitive approaches, teachers can use scaffolding (including modelling or direct instruction, and sequential questioning) and training on the problem solving process as described by this study's proposed model.

Cognitive load reducing aids in mathematics were found by Hattie (2009) to have a *d* statistic of 0.45. The importance of reducing cognitive load is very important, as available cognitive resource is one of the main factors that can allow or stop a solver from solving a problem. Reducing cognitive load permits an increase in the availability of cognitive resources. There are many cognitive load reduction strategies that the participants have yet to consistently employ.

First, there was a tendency for certain students not to use pen and paper. Students must understand the limitations of their cognitive architecture and realise that trying to work out complex calculations in their head will likely lead to mistakes and may not always be a productive use of cognitive capacity. Pen and paper serves as an extension of working memory. If there is ever a need to remember an important piece of information, writing it down instantly allows the student to clear space up in working memory for additional information. Often, the point at which certain crucial information was written down was the point at which clarity began to appear.

Second, most participants were not very good at transforming or restructuring problem forms. In mathematics, it is crucial to be able to represent a problem in a more fundamental form. For example, an angle sum of a triangle word problem could be translated to a diagram. The diagram could then be translated to an algebraic equation. Each of these states is more fundamental than the other. Specifically teaching students to translate questions from one form to another would be a general skill that is transferrable to many problem types.

Third, students should be taught to accurately monitor their cognitive load. When interviewed, the participants stated that some questions made them feel "overwhelmed" – a description of cognitive overload. It is at this time when students need to recognise their overloaded state of mind, reorganise their thoughts and approach the question again from a fresher perspective.

Modelling and direct instruction are ways of helping students to develop rich schemas necessary for problem solving. Modelling involves an expert problem solver demonstrating the

exact sequence of steps required to solve a particular problem type. In modelling, a teacher says, "here is a problem, and this is how I solve it." Each step of the problem solving process is explained so that students know what the teacher is doing and why they are doing it.

Direct instruction was given a *d* score of 0.59 in Hattie's (2009) study, indicating a link of moderate strength between direct instruction and student learning. In terms of the present study, modelling of certain parts of the solution path was found to be an effective means of aiding in the reduction of cognitive load and the achievement of clarity.

The use of sequential questioning is another technique that can be used as a scaffolding technique. It was given a *d* score of 0.46 by Hattie (2009). In this study, sequential questioning was useful as a form of guiding the student towards the correct answer. Another form of instruction that models solutions is worked examples. Worked examples were found to have a *d* statistic of 0.57 in Hattie's (2009) study.

In Hattie's (2009) study, teaching for problem solving was found to have a *d* statistic of 0.61. From the proposed model, it can be seen that; firstly, reading, internalising and classifying; secondly, mental restructuring; and finally, solution strategies are three problem solving tactics that affect cognitive load. If students know that these things happen, they have the opportunity to control them. They are able to say to themselves, "Okay, I've tried reading the problem and some solution strategies, maybe now I need to try some mental restructuring." It is at this point that an understanding of metacognition is valuable. Teaching metacognitive strategies was found by Hattie (2009) to have a *d* statistic of 0.69, which falls in the moderate to strong range. Teaching self-verbalisation/self-questioning, which was strongly encouraged

in this study and is essentially metacognition spoken aloud, was found by Hattie (2009) to have a *d* statistic of 0.64.

5.4.1.2 Psychological Approaches

In the proposed model, psychological approaches such as prior achievements, motivation, concentration and persistence, feedback, and teacher clarity are encompassed within with the internal or external factors. These aforementioned strategies are classed as psychological because, while they may have a cognitive aspect to them, they affect the emotional realm as well.

In Hattie's (2009) study, prior achievements were found to have a *d* statistic of 0.67. This fits in with the view that self-efficacy and confidence have a role to play in problem solving. Teachers should ensure that students have a stock of positive prior achievements upon which to draw. This can be difficult to ensure as a negative prior achievement can outweigh many positive ones. Students could be taught Thomas' method of solving questions in the order that gives the greatest confidence for later questions. Confidence is something that can be, to a certain extent, manipulated by the student.

Motivation was an area that, while not directly referred to in this study – although there was some related discussion in the final interview session – is closely related to task significance, which was referred to directly. They are related because if a student sees a task as significant, they will be more motivated to do better in it. Hattie (2009) found the *d* statistic for the

relationship between motivation and learning was 0.48. It is partly the teacher's role to motivate and enthuse students. Indeed, in Hattie's (2009) study, teacher-expectations and teacher-student relationships have *d* statistics of 0.43 and 0.72 respectively. The most desirable outcome would be for teachers to instil self-efficacy and intrinsic motivation in the students by giving them the opportunity to enjoy success in mathematics. Teachers can put the students' brain's inbuilt reward circuit to good use by providing skilfully graded problems which develop success over an array of problems of gently increasing difficulty – a staggered instructional approach. However, if encouraging intrinsic motivation proves difficult, external motivation still has its place in the mathematics classroom.

Concentration and persistence were found to have an effect of 0.48 according to Hattie's (2009) results. It is important to encourage persistence in students as persistence is one means by which cognitive confusion can be resolved. Persistence can be encouraged by the fostering of intrinsic motivation or the offering of external rewards.

Finally, feedback and teacher clarity were two strategies I depended on heavily during the interview sessions. Giving immediate feedback to the students while they were solving the problem gave them confidence to try things, knowing that I would be there as a safety net for them. This feedback assisted the management of their cognitive load because I told them whether they were on the right path or not. If they were not, they could discard that solution path, thereby reducing cognitive load by reducing the number of potential paths that could be taken. If they were right, then they could focus their thoughts on the new, more correct, path. Feedback also allowed the participants to check the validity of their solution paths. Positive feedback positively reinforced their efforts and strengthened that particular solution schema

without fear of having made any mistakes. Teachers should similarly ensure that students are aware of which schemas should be reinforced and which should be de-emphasised, by giving timely feedback. If the participants in this study did require scaffolding, I aimed to be as clear as possible with my explanations. Clarity on the part of the teacher reduces the initial cognitive load produced during schema acquisition as the learner does not have to spend additional resources figuring out what is useful and what is not. Clarity and feedback were found by Hattie (2009) to have *d* statistics of 0.73 and 0.75 respectively, showing a moderate to strong correlation with student learning.

5.4.1.3 Staggered Instructional Approach

One of the principal premises of cognitive load theory is that as cognitive load increases, question difficulty increases for the particular individual. Teachers can take advantage of this phenomenon by producing an artificial gradient of question difficulty through manipulating intrinsic cognitive load. This can be put into place within the framework of worked examples (Sweller, 1999). First, teachers can use direct instruction to teach the required concept. They can then use expert modelling to further explain the problem solving process. At this point, teachers can choose to introduce fully worked examples. These examples show every step in reaching a solution from a problem state. Students can refer to these as they build up a rich schema about their topic of study.

As the students gain expertise, some of the helping factors can be removed. Worked examples become partially worked examples, where students have to fill in certain steps in a question. This then allows a gradual increase in each question's intrinsic cognitive load. Towards the end of this teaching process, teachers may pose questions with little resemblance to the original

question upon surface examination, but sharing the same underlying structure. Giving students familiar questions under a different guise can expand and improve their solution schema for that particular kind of question. This also builds students' abilities to identify underlying surface features within diverse problem forms and use problem restructuring to convert these questions into familiar forms which can be solved using familiar schemas.

A case must be made for specific training on the transformation of questions to other forms. It is absolutely crucial that students understand the restructuring strategy as it is a necessary step in many problems. Students should be trained in recognising when a question needs transforming and have an understanding of the process of manipulating the form of the question. This is a skill which is not domain specific, so investing a small amount of time in this could prove valuable in the long term.

Finally, it is important to see that a mathematics problem can consist of a long series of steps. Each step usually depends on the step before. Therefore, if a student cannot work out a particular step, they may be unable to complete the problem even though it was only a single intermediate step that was holding them back. Since scaffolding is not allowed within tests, it is proposed that a teacher could set up a system whereby if a student is unsure of how to complete a certain step, they could use a pre-defined value to start the next step with. For example, suppose that one section of a problem yields the answer 25, and the next section needs this number 25 as input data to continue processing. If the student is unable to calculate the required number 25, they could write "I don't know what the answer to this step is but let the answer to be used for the next section be 10." This would allow students to show more of their mathematical skill without seriously compromising the question.

5.5 Limitations of the Study

This study had a number of limitations that must be addressed. First, as the interviews had to take place at times convenient for each of the schools, the four sessions were all limited in duration and regularity. This led to time pressures during the sessions which led to premature scaffolding and a loss of detailed questioning opportunities. One participant was not able to complete the algebra session due to consistent illness. Second, this study was limited to a group of six students from private Christian schools. Further, these students were chosen because of their higher mathematical ability. However, these students could not be classified as mathematically gifted. Hence, the results really describe cognitive load factors in relation to good average students. The homogeneity of the groups reduces the ability for these results to be generalised. Nevertheless, this study has provided explanation of problem solving processes and has presented a model of problem solving from the perspective of cognitive load theory.

5.6 Recommendations for further research

A number of further research possibilities arise from the present study. The role of metacognition in monitoring and reporting cognitive load is a possible area of further inquiry. In particular, further research could focus upon the relationship between cognitive load and metacognitive function. In addition, further study could be undertaken on the how the internal and external factors impact problem solving methods. In particular, the effect of self-efficacy and confidence on perceived and actual cognitive load would be an interesting area of inquiry. A deliberate and specific study on the effects of scaffolding on cognitive load and on the way scaffolding impacts the problem solving methods during the solution stage of problem solving could be another area of future research. Finally, an expanded study with a greater

number of interview sessions and students from diverse mathematical backgrounds would partially resolve the limitations outlined in the previous section. A study to confirm or deny the proposed model of problem solving, regardless of the research method used, would also serve to refine our understanding of the effects of cognitive load during problem solving.

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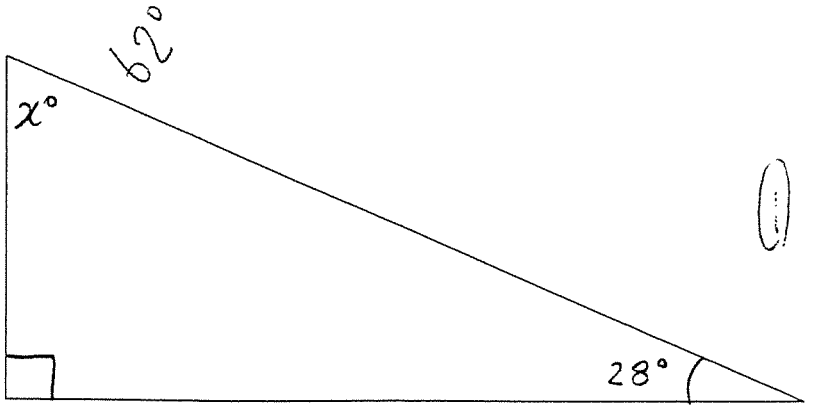
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Appendix A

Geometry Session Question Booklet (Sample)

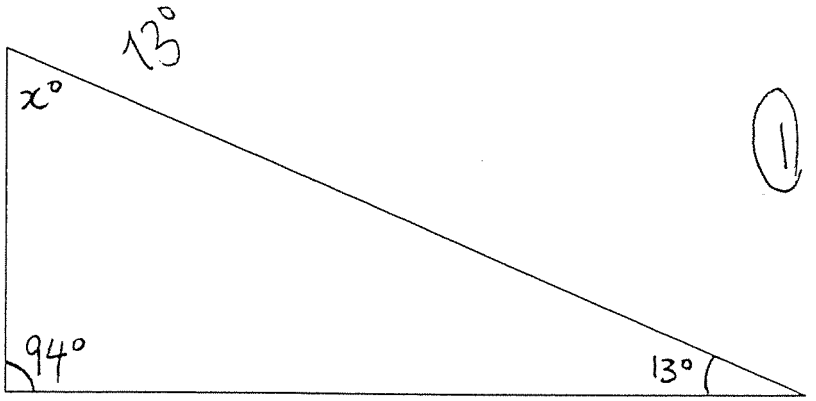
AVONDALE COLLEGE LIBRARY

Name: [redacted] Date: 19/6/09



(1)

1. Find the value of *x*.

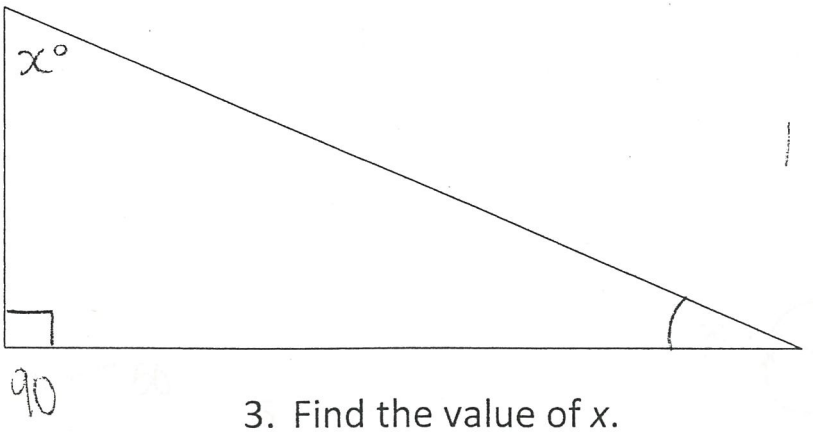


(1)

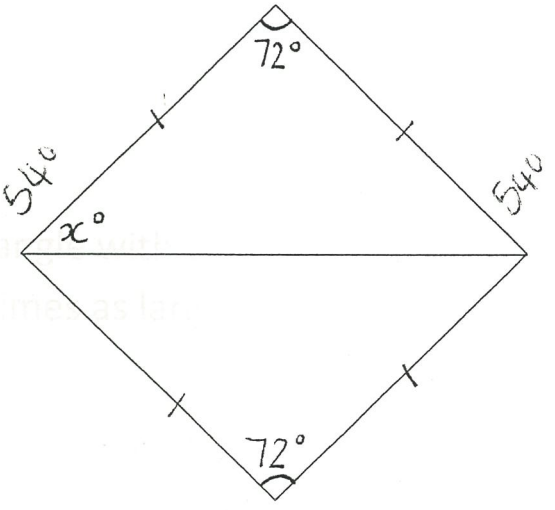
2. Find the value of *x*.

0

Name: Date:

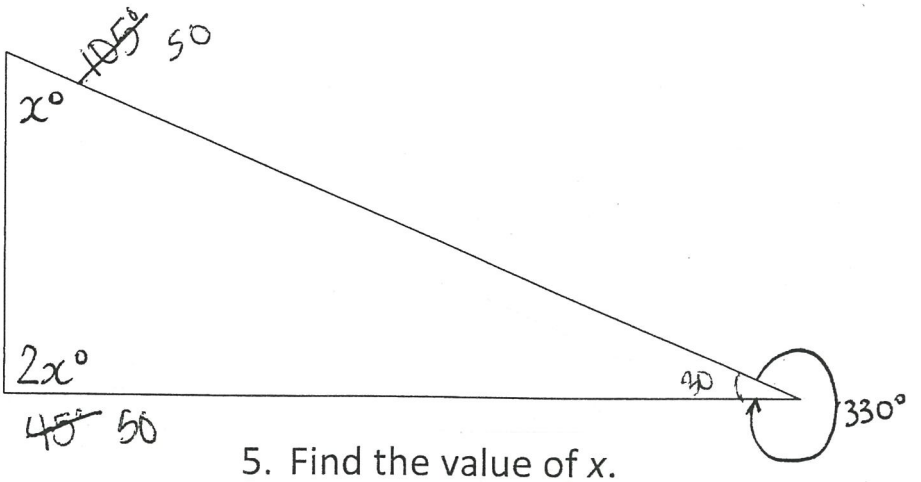


3. Find the value of *x*.



4. Find the value of *x*.

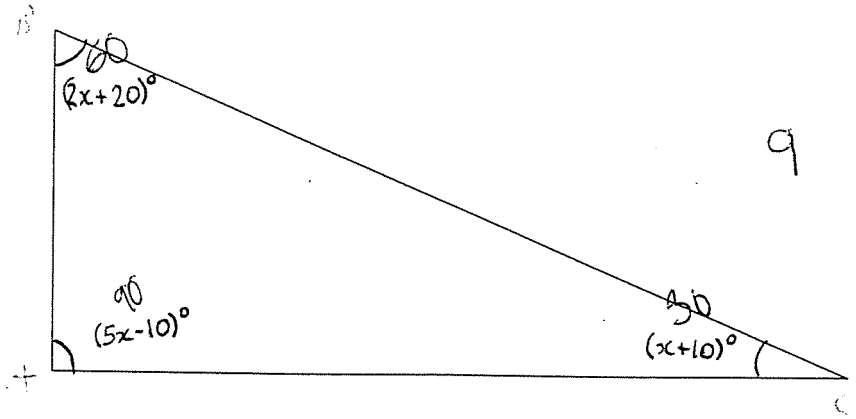
Name: Date:



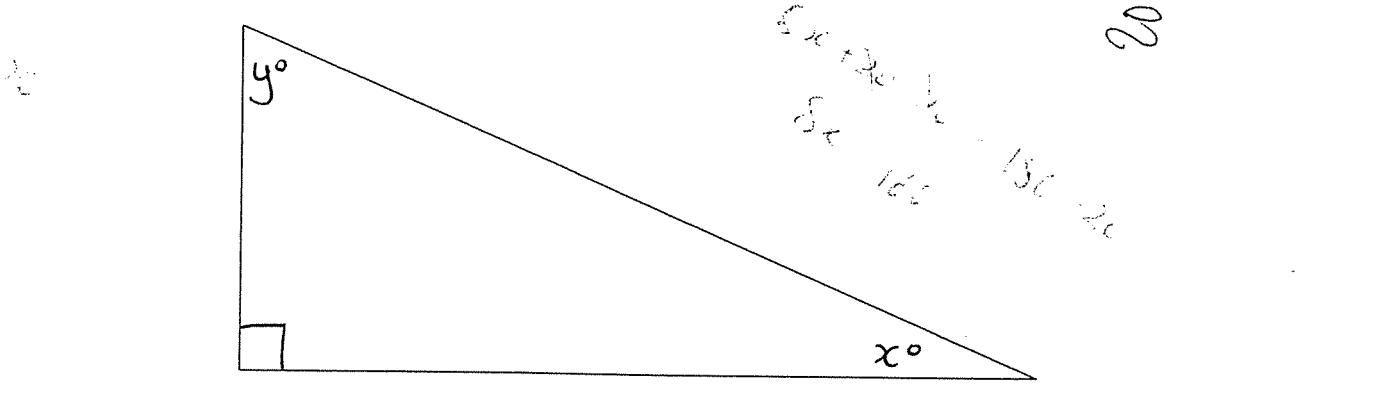
5. Find the value of *x*.

6. There is a triangle with 3 angles; A, B and C. Angle A = 90°. Angle B is 3 times as large as angle C. Find the size of angle C.

Name: Date:

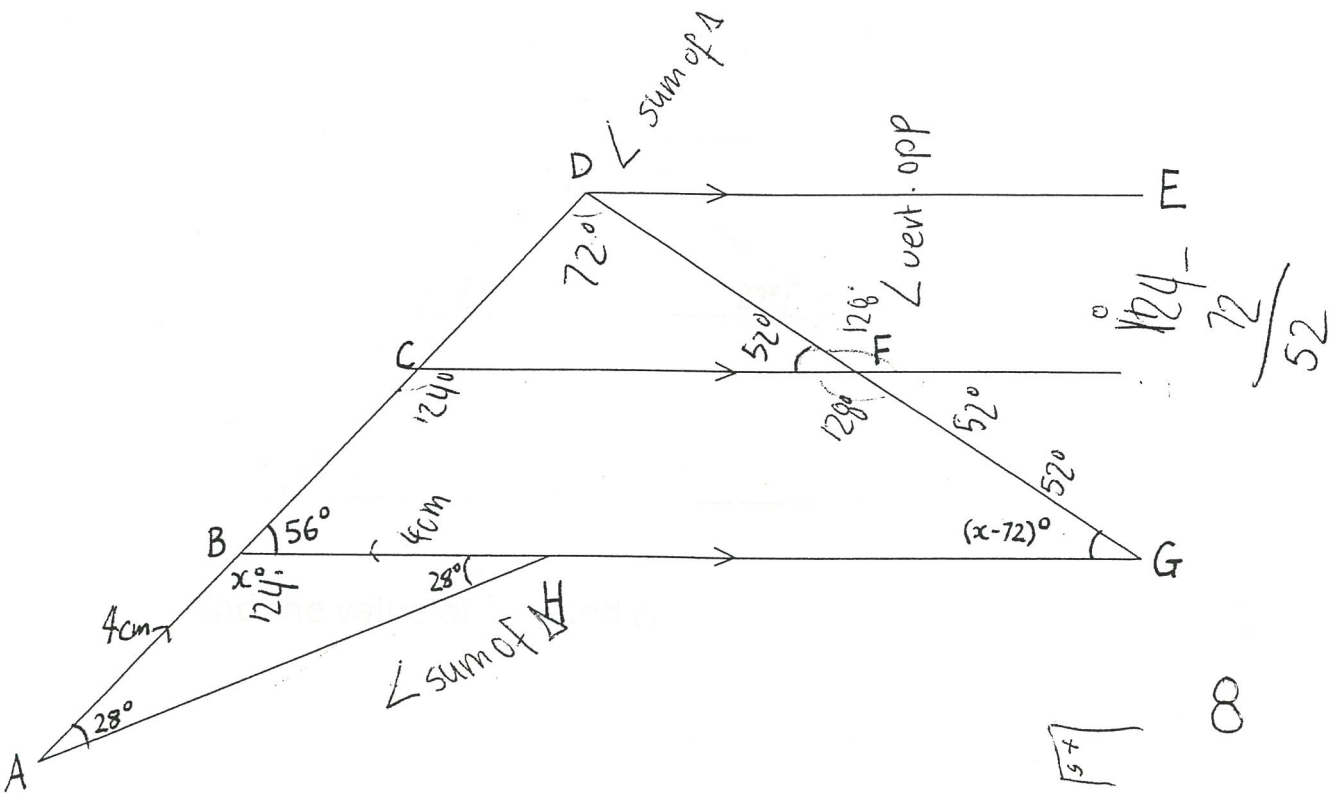


7. Find the value of x.



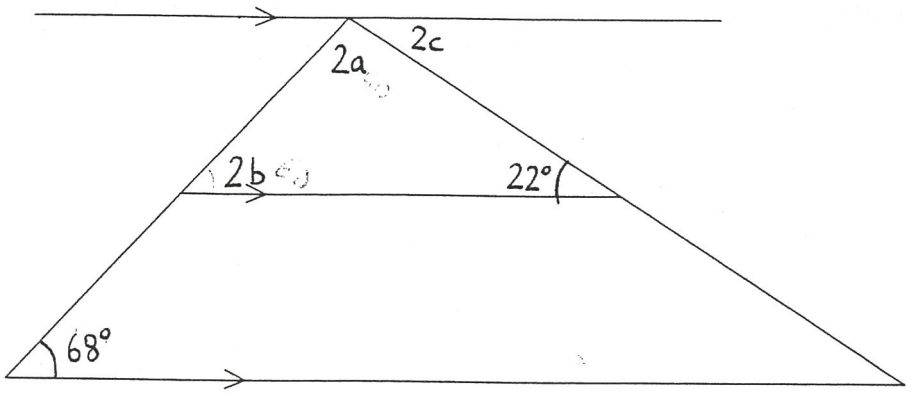
8. In two different ways, prove that $x + y = 90$.

Name: Date:



9. Find the value of as many angles and sides as you can, giving reasons for each. You may calculate the angles and sides in any order.

Name:

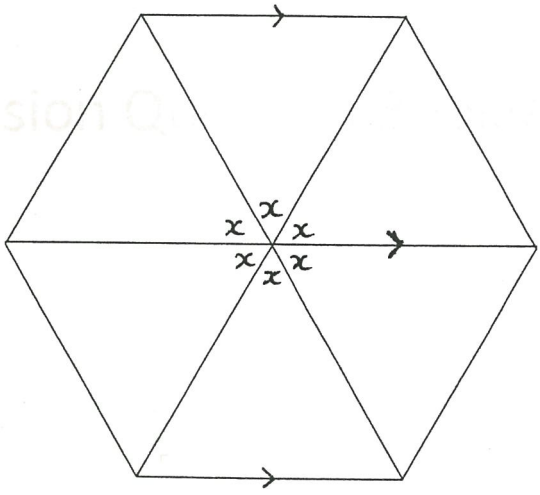


10. Find the value of a, b and c.

11. Prove that the exterior angle of a triangle is equal to the sum of the interior opposite angles.

Name: Date:

12. Prove that the diagonals of a square bisect each other at 90°.



13. Find the value of as many angles and the lengths of as many sides as possible.

Appendix B

Algebra Session Question Booklet (Sample)

Name: [redacted] Date: 1 1/9/09

2

1. The RSPCA rescues, on average, 16 animals a day. How many animals does the RSPCA rescue in a week?

2

2. How many days would it take the RSPCA to rescue 176 animals?

f

3. When $x = 5$, $y = 8$. When $x = 7$, $y = 12$. When $x = 6$, $y = 10$. What is a possible relationship between x and y?

x	5	6	7
y	8	10	12
2x	10	12	14

$y = x$

$y < 2x$

$y = 2x - 2$

f

4. $x = 12$. $2x = 3y$. What is the value of y?

Name: Date: 2

5. $2x = 3(x - 2)$. Find x. (3)

$2 \times x = 3(x-2)$

$2x = 3x - 6$

$-x = -6$

$x = 6$

(6)

6. A man buys 2 cows and 18 chickens. The amount of money spent on cows is the same as the amount of money spent on chickens. In total, the man spends $3060. What is the cost of 1 chicken? What is the cost of 1 cow and 2 chickens?

$935.00 $85.00

$3060

cows $1530 — $765 p cow

chickens $1530 — $85 p chicken

$1 = 9h

680

$$9\overline{)3060}\quad 340$$

$765.00
$170.00
$935.00

2x

(3)

7. A certain number multiplied by two gives you the same answer as firstly taking away 2 from the number and then tripling it. What is the number?

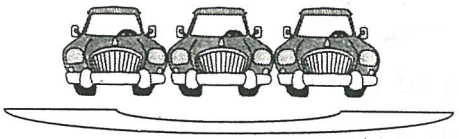
$-2x$

$2x = (x-2) \times 3$

$= 3(x-2)$

Name: Date: 3

8. See diagram:



A company donates $2000 to charity for every 3 cars sold

$3c = 2000$



The company has a maximum of $42000 available for donations



x 57

The company has 57 cars in its caryard. Can it afford to pay the charity if all 57 cars are sold?

19 x 2000 63 yes it can afford it. $3800.00



What is the biggest number of cars the company could sell and still be able to afford to pay the charity? 63 cars

Name: Date: 4

8

(Text for Question 8) – For every three cars that a company sells, it offers to give $2000 to charity. The company has set aside $38000 for this charity fund and it cannot afford to pay any more. Will the company have enough money to pay the charity if all 57 cars in the caryard are sold? How many cars is the maximum the company can afford to sell?

P X

8-9

9. A boy adds together the number of PS3 games and Xbox 360 games he has. He has 8 games in total. He then multiplies the number of PS3 and Xbox 360 games he has and finds that they multiply to 12. How many of each kind of game does he have? He has 4 more PS3 games than he has Xbox games.

$p + x = 8$

~~$4 \times 3 = 12$~~

$p = 4 \times x$ or



① $x + 4 = p$

② $x + p = 8$

$x * p = 12$

$x + (x + 4) = 8$

$2x + 4 = 8$

$2x = 4$

$x = 2$

$x 4 x p = 12$

Appendix C

Coding Categories Sample

ession 2 2,7,9

*41:14 Using trial+error working off p-4=x eqn. Then confirming with the other eqns.

*38:24 Struggling to figure out what bit means so he re-wrote what seemed to be the most important.

50/53

Categories

Crude, but not bad for Yr. 8.

roblem identification

- Problem type recog./schema activation
- Reading of problem
- Strategies to organise solution
  - Writing relevant information
  - Filter superfluous info/distractors
- Problem form

raversing solution path

- Schemas/means-ends/trial & error
- Transformation of question
- Identifying patterns
  - Mental visualisation
- Wrong/backtrack/metacognition
- Systematic
- Breaking problem up

ognitive load

- Cognitive overload
  - Recognition
  - Reduction
  - Leaving it and coming back
- Scaffolding
- "Oh yeah"/"You just see"

ediating performance issues

- Confidence/self-efficacy
  - Previous task performance
- External factors
- Perseverance
- High confidence
  - Solution speed/Error rate
- Low confidence
  - Guessing
  - Giving up
  - Comparing with others

3:30 Letters activate algebra schema.

2:37 *11:00 knew what to do straight after reading.

22:36 Looks easy at first but keep reading and it gets harder.

26:49 *22:50 Identify goal and represent with x.

*20:16 Write down price of 1 cow so you remember it. This is valuable info.

6:14 The only one to put x's in order in the table.

Q5 Didn't use algebra. Lack of expertise.

*35:35 Text version of Q8 harder. Worth quoting.

27:20 Change car into x, but probably not the best approach.

14:50 "The wave" for expanding brackets. He decided not to do that.

Q1 Schema-based
Q2 Clear explanation and solution.
Q2 in head.

See 24:40 side of page.

Q4 Definitely schema-based "You gave me all the information I needed to find out what steps I need to do"
Q5 Used trial-and-error but wrote working down.

Q9 T&E

4:54 x↑1 y↑2
23:57 Saw link with Q5.
+8:43 Relationship was more obvious when I asked for equation form.

22:52 Represent number as x and build required equation while reading the question. Then solve (or link to Q5).

11:47 Asking [redacted] to break it up into steps. This achieved more clarity.

*21:44 Remembering cost of cow vs. writing it down.
"It would've made it more frustrating because I'd have to put one aside in the left side of my brain and I'd have to work it out with only ½ my brain so I'd be thinking less".

Great insight.

"English words have more meaning than maths does"

these happen when cognitive load is high. First time it happened in this ~~video~~ audio was 39:11.

39:11 (Explanation 39:33) *39:45

11:43 "Does that give you enough information?"

Q6 Solved ingeniously. 18 chickens = 2 cows ∴ The equivalent of 36 chickens were bought. This bit of working shows how [redacted] interpreted the "same amount of money..." line. He then found the cost of 1 chicken (3060/36) to solve part 1, and $85x9 + $85x2 for part 2.

*24:10 Discussion of strengths and weaknesses of T&E vs. schema-based solution paths.

*Words vs. algebra 24:40 [redacted] found numbers only easier. He knows it's a personal preference. In problem solution, is it worth turning an algebra question into a word problem if you're a word person, or should everyone turn words into algebra and solve it that way?

*25:36 Why did you find the word one harder?

29:07 How do you know what to do, and yet you can't explain it? He would be better at explaining maths questions than English ones.

*30:40 First, find donation per car. This works, but [redacted] 2000/3 is a messy answer. He "put this to a side because there's a lot of ways to work out this answer and this is just one way, but it has a barrier". "You work out the answer another way, then come back to this one and learn from it". There was a lot

Appendix D

Student Information Sheet

Avondale COLLEGE™

Student Information

My name is Nicolas Zenteno and I am currently completing a Bachelor of Education (Honours) at Avondale College. As part of my Honours program, I will run a study into problem solving in Mathematics which will eventually be written up as a thesis.

The study is about how students use diagrams to help them solve Maths problems. Specifically, some problems are too difficult to be solved in your head, so you need to create a diagram on paper to see all the information you need to know in order to solve a problem, right there in front of you. This study examines how students construct these diagrams in an attempt to reduce the heavy load these problems place on working memory.

The first thing I will get you to do is to take a mathematically-related patterns test called Raven's Progressive Matrices. After that, I will spend time with you as you solve problems. This will be one-on-one work, but the teacher will always be present in the room. You will be asked to work through Maths problems, talking about the way you solve them and especially about how you go about constructing a diagram. I will ask you questions about how you are going with the problem, and after we finish, we will have a discussion about how you answered the question. It is expected that this process **may** help you in Maths in a similar way to how coaching would. All sessions spent together will, with your permission, be audio-recorded so that later I can refer to what you said.

You can interrupt or withdraw from the study at any time without penalty. If you wish, you can also ask questions related to Maths and problem solving.

Information arising out of this study will remain confidential and the only people who will see your work will be your teacher, my supervisor and I. Attached you will find a consent form. If you would be willing to participate in my study, please complete and sign the form and return it to me through your class teacher.

Thank you and sincerely,

Nick Zenteno
Honours student
Avondale College

Appendix E

Parent Information Sheet / Consent Form

Avondale COLLEGE™

Research Information Sheet

My name is Nicolas Zenteno and I am currently completing a Bachelor of Education (Honours) at Avondale College. For the Honours component of my degree, I am conducting research under the supervision of XXXXXXXX in the Faculty of Education. I would like to gain your informed consent so that your child may be included in my study.

I would like to perform a study examining how able students internally represent a Mathematics problem as a way of reducing cognitive load. This will take the form of giving participants questions from the topic they are currently studying in Mathematics, and asking them to verbalise the process they undertake in solving the questions. In particular, there will be an emphasis on how they construct diagrams as mental representations of the problem. **It is anticipated that there will be between four and six one-hour sessions with each participant. These sessions will occur at a mutually agreed time that is suitable to the school and is likely to be within school hours or to occur immediately at the close of school.** With the permission of the participant, the sessions will be **audio-recorded** and transcribed.

Upon parental and participant consent, no more than six participants will be selected in consultation with their class teacher. These participants will be given a Raven's Progressive Matrices test, a pattern-based test that provides a measure of working memory efficiency. All participants will be asked to work through the same given problems. This work will be one-on-one. The venue will be a school classroom and a teacher will be present at all times. I will also talk with the participants as they solve the problem, examining their strategies and suggesting additional methods of solving problems. It is expected that participants will benefit from these exchanges as they would from conventional coaching. **Participants will be free to withdraw from the study at any time without penalty and without having to provide a reason.**

This research project has been approved by the Avondale College Human Research Ethics Committee (HREC). Avondale College requires that all participants are informed that if they have any complaint concerning the manner in which a research project is conducted it may be given to the researcher, or if an independent person is preferred, to the College's HREC Secretary, Avondale College, PO Box 19, Cooranbong, NSW, 2265 or phone (02) 4980 2121 or fax (02) 4980 2117 or email: research.ethics@avondale.edu.au

All data will be confidential and the only people who will see the participants' work will be their teacher, my supervisor and I. The names of participants and the name of the school will not be revealed in my thesis or in any report or publication that may arise from the study. The thesis will be held by Avondale College and any report or publication will be made in venues or journals associated with educational research. Attached you will find a consent form. If you would be willing to permit your child to participate in this study, please complete and sign the form and return it to the school with your son or daughter. Final selection of the participants will be made in consultation with the teachers of the school.

Thank you and sincerely,

Nick Zenteno
Honours student
Avondale College

Appendix F

Sample of Interview Session Questions

Do your best to describe to me, in general terms, how you solve a problem.

Example: I read it through until I understand everything clearly, then I look for all the important information and write it down below the problem, and then I try to take steps so that every step I take moves me a little bit closer to the answer.

What happens when you get a problem that is easy to solve?

Example: As soon as I look at it, I know what to do straight away so I just solve the problem without really thinking about what I'm doing.

Do you feel like you can see the entire solution in one go with easy/middle/hard ones?

Example: With the easy ones, yes. With the middle ones, I can only really see one step in advance. With the hard ones, I have trouble even seeing what I'm meant to be doing for the step I'm on.

What happens when you get a problem that is hard to solve? Are there any strategies you use?

Example: I read through the problem again and again until I feel like I understand enough of it to write down the important stuff. If I don't know enough of what to do, I just give up. The strategies I use are reading the question over and over again and highlighting the important parts of the question in my head. I also write things down.

What happens when you get a problem that just the perfect difficulty for you?

Example: I look at it and I kind of know what to do but kind of don't. When I write down my information, every step seems to come from the step before, but I can never really see more than one step in advance.

Can you tell me anything about what happens when you're working out a problem and suddenly just go, "oh yeah!"?

Example: All of a sudden, I just know what do. It's as if all the different stuff going around my head that was really disorganised just clicks into place, like a jigsaw puzzle.

When you see that a question has a lot of information in it, what do you do to organise the information in your head?

Example: I try to break up the problem and solve one bit at a time. To do this, I try and get rid of any information that isn't really needed to solve the bit of the problem that I'm concentrating on. I then repeat that until I solve the whole problem.

Do you monitor your own thinking when solving a problem? For example, do you ever think to yourself, e.g., "What's this question asking me? Oh yeah. OK, now I need to do this... OK, I've done that, hmmm, let's see... Oh yeah, I'll do this next".

What does your internal dialogue sound like?

Do you find it's different for easier, middle and harder questions?

Appendix G

Table of Subjective Cognitive Load Ratings

Appendix G: Table of Subjective Cognitive Load Ratings

GEOMETRY	Matthew	Thomas	Rosie	Jake	Wendy		Question Mean	Question Std. Dev.	Difficulty Rank
1	1	2	1	2	1		1.40	0.55	2
2	1	2	1	1	1		1.20	0.45	1
3	1	4	2	5	1		2.60	1.82	3
4	10	6	3	7	1		5.40	3.51	5
5	1	7	9	4	9		6.00	3.46	6
6	1	7.5	8	5	10		6.30	3.46	7
7	7	7	6	N/A	9		7.25	1.26	9
9	7	8.5	6	6	8		7.10	1.14	8
10	5	N/A	N/A	N/A	N/A		5.00	N/A	4
Mean	3.78	5.50	4.50	4.29	5.00		4.60	0.66	1 = Easiest
Std. Dev.	3.53	2.52	3.16	2.14	4.31	Overall Measures	3.13	0.85	9 = Hardest
Rank	1	5	3	2	4				

ALGEBRA	Matthew	Thomas	Rosie	Jake	Wendy		Question Mean	Question Std. Dev.	Difficulty Rank
1	1	2	1	1	1		1.20	0.45	1
2	1	2	3	2	1		1.80	0.84	2
3	3	4	10	4	8		5.80	3.03	9
4	1	4	6	3	1		3.00	2.12	3
5	4	3	5	3	6		4.20	1.30	4
6	8	6	4	3	5		5.20	1.92	6
7	5	3	9	2	9		5.60	3.29	8
8A	2	7	6	2	10		5.40	3.44	7
8B	2	4	6	2	10		4.80	3.35	5
9	4	8.5	6	5	10		6.70	2.49	10
Mean	3.10	4.35	5.60	2.70	6.10		4.37	1.49	1 = Easiest
Std. Dev.	2.23	2.16	2.63	1.16	3.90	Overall Measures	2.42	0.99	10 = Hardest
Rank	2	3	4	1	5				

Note: No data is available for Hannah (see §3.5.3).