Mathematical manipulatives: Creating an environment for understanding, efficiency, engagement, and enjoyment

Ashlee Cockett
Graduate, Avondale College of Higher Education, Cooranbong, NSW

Peter Kilgour
Lecturer, School of Education, Avondale College of Higher Education, Cooranbong, NSW

Abstract

This study examines the impact of using manipulatives in Mathematics on student understanding, efficiency, engagement and enjoyment in a lower primary classroom. Manipulatives are concrete physical objects that students can use in a hands-on approach to learning. During this research, 32 students were involved in various mathematical activities involving different kinds of manipulatives and also in activities where manipulatives were not used at all. The types of manipulatives used were clocks, coins, MAB blocks and the interactive whiteboard. Quantitative data was collected using a survey while qualitative data in the form of observations was also collected. The results showed that students are more engaged when using manipulatives, and that their perception of their learning environment improved in the areas of enjoyment, understanding and efficiency.

Keywords: manipulatives, kinesthetic, learning styles, technology, concrete, virtual

Introduction

This paper reports on the impact of using manipulatives in Mathematics for developing mathematical concepts and on student perceptions regarding the use of manipulatives. More specifically, this research project investigates whether using manipulatives in conjunction with worksheets is more beneficial for students' understanding, engagement, efficiency and enjoyment of Mathematics than using worksheets alone. For the purpose of this study, student engagement is the extent to which students become involved in the learning activities in the classroom, and efficiency refers to the process of problem solving and methodically arriving at solutions.

The term 'manipulatives' is defined as, "Items that students use to support hands-on learning. Manipulatives (such as markers, toothpicks, or coins) provide visible models that help students solve problems and develop concepts" (Lewis, 2012, p. 1). Visual manipulatives include technology such as interactive whiteboards and computer games that are two-dimensional representations of three-dimensional space.

In recent years, especially in Mathematics, many have questioned whether using manipulatives is beneficial and some have suggested that manipulatives are over-rated as a teaching resource (Uttal, 2003; McNeill & Jarvin, 2007). Manipulatives can range from blocks and plastic dinosaurs to counters and the use of interactive whiteboards. Research suggests that manipulatives can help students develop Mathematics concepts in a concrete form that is visual to the learner (Swirling, 2006).

Background

There has been an increase in the use of manipulatives, also called concrete materials, in the teaching of Mathematics (Boggan, Harper, & Whitmire, 2010). A mathematics manipulative is described as a concrete or visual object that allows a student to explore Mathematics concepts using a hands-on and active approach. These objects can include blocks, shapes, cubes, money, counters or even paper (Mathematics A tube, 2012, p. 1). The 21st Century provides students with a variety of manipulatives including virtual manipulatives. "[Virtual manipulatives] are basically digital 'objects' that resemble physical objects and can be manipulated, usually with a mouse, in the same ways as their authentic counterparts" (Cited research centre, n.d., p. 1). Virtual manipulatives, which are usually modelled after concrete manipulatives, are often web based. Bouk and Flanagan's (2010, p. 187) study found that "students can perform almost any function with a virtual manipulative as they could with a concrete manipulative". Examples of virtual manipulatives can be found in computer games and on interactive whiteboards.

Manipulatives make learning meaningful

Manipulatives can be an important tool to help students think and reason in a more meaningful way. Stein and Bovalino (2001) concluded that by providing manipulatives, teachers create a more meaningful experience for students by offering a concrete form for which students can then see the relevance.

Manipulatives aid engagement and understanding

Manipulatives are able to facilitate the creation of a learning environment that encourages engagement and enables understanding. Florence (2012) argues that mathematics manipulatives can help engage students for a longer period of time by helping them stay focused on particular tasks. She believes that lecture based teaching can often seem boring but that manipulatives allow students to be actively involved in learning. Xie, Antle, and Motamedi (2008) linked enjoyment and engagement in their study of the use of tangible objects in the learning process. A study by Swirling (2006) showed that the use of concrete or virtual manipulatives could improve students' learning when dealing with complicated concepts. It was found that when manipulatives were used effectively, student understanding and engagement increased. Moyer (cited by Bouk & Flanagan, 2010, p. 187) believes the benefits of virtual manipulatives include facilitating the introduction or revision of Mathematics ideas, aiding the understanding of visual concepts through the use of visuals, scaffolding learning, and engaging students in learning.

Using manipulatives in Mathematics increases the students' confidence to complete difficult mathematics problems. Shaw (2002) suggests that many children see Mathematics as a struggle so they give up on the task. Shaw also suggests that the use of manipulatives can counter this. "When students physically move manipulatives to show various relationships, their sense of touch is actively engaged" (p. 3). This works to engage the kinesthetic side of the learner, thus aiding understanding.

Manipulatives cater for individual needs

Manipulatives can be a useful tool to cater for different learning styles and are particularly appropriate for kinesthetic and visual learners (Sundstorm, 2012). Kinesthetic learners learn best by physically touching objects and playing with them. Mathematics manipulatives allow children to handle objects in order to get a real representation of mathematics concepts. Mathematics manipulatives for visual learners can include flash cards or posters that allow students to gather a clearer understanding of the mathematics problems (Sundstorm, 2012).

Manipulatives can also be used to cater for individual learning needs, particularly for those students who tend to struggle with mathematics concepts. Some students need to use concrete materials to learn how to count, while other students need manipulatives to increase their understanding of place value. Research indicates that using manipulatives is especially useful for teaching low-achievers, students with learning disabilities, and English language learners (Boggan, Harper & Whitmire, 2010).

Manipulatives aid the movement from concrete to abstract

The key objection to the use of manipulatives is that: "Children have trouble linking representations based on manipulatives with written, symbolic representations" (Uttal, 2003, p. 4). Uttal found that students could use either written form or manipulative form but could not combine the two to gather meaning. He discovered that "they often could succeed with manipulatives or with written representations, but they failed to connect the two" (p. 4). McNeill and Jarvin (2007, p. 1) suggest that

this problem arises when the “teacher fails to explicitly make the link to their Mathematics purpose in the activity”. This highlights the important role of the teacher in helping the child to make connections.

In contrast, Boggan, Harper and Whitmire (2010) suggest that using manipulatives in Mathematics is beneficial for students’ learning. They found that “manipulatives help students learn by allowing them to move from concrete experiences to abstract reasoning” (p. 4). Manipulatives are an effective teaching tool because teachers can use them to teach students how to bridge the gap between concrete and abstract learning (Hawkins, 2007). However, the timing of the movement to abstract learning is important. “The use of manipulatives enhances concept formation when both the concrete and the connecting stages are fully understood before moving to the abstract” (Kentucky Center for Mathematics, 2012, p. 1).

In summary

To make maximum impact, the use of manipulatives needs to be carefully planned. Smith (cited by Boggan, Harper & Whitmire, 2010, p. 3) states, “There are probably as many wrong ways to teach with manipulative as there is without them”. They also advocate manipulatives that are appropriate for the learners and chosen to meet the specific goals and objectives of the lesson.

Mathematics manipulatives can be an important tool for helping students to develop Mathematics ideas. The benefits are associated with the increase in student engagement and enjoyment, which leads to increased understanding and efficiency. Manipulatives can be in the form of visual objects that students can touch and play with or virtual manipulatives that can be seen through technology. Either way, manipulatives are able to cater for a variety of learning styles and abilities within classrooms. Although there is some research suggesting students cannot connect the manipulatives to the written form, the dominant finding of the research is that manipulatives are beneficial for developing mathematics concepts.

Method

This research project was conducted over a six-week period in a single class of lower primary students. All students in the class were invited to participate.

A mixed method approach (Onwuegbuzie & Wisdom, 2014) allowed data to be collected in a variety of ways. While students were completing their work, anecdotal notes were made of students’ engagement, enjoyment and productivity in lessons both with and without the use of mathematics manipulatives. During the study period, work samples were collected from several lessons, including lessons conducted with and without manipulatives. These samples were evaluated in order to determine whether students’ learning benefited from the use of manipulatives. A questionnaire was also used to assess students’ perceptions regarding whether they enjoyed using manipulatives and whether they found them effective when completing Mathematics activities. Students were asked to respond to the following prompts using a 4-point scale (1: Never, 2: Sometimes, 3: Usually, 4: Always):

> I enjoy using objects during Mathematics lessons.
> Mathematics objects help me understand Mathematics better.
> Mathematics objects make me finish work quicker.

Lessons taught with the aid of manipulatives were introduced using the interactive whiteboard to allow an opportunity to engage in activities that required moving and touching the board. As the lesson developed, manipulatives were used in combination with worksheets, to enhance understanding of mathematical concepts. The lessons concluded with a manipulative being passed around while students reflected on what they had learnt.

Lessons taught without the aid of manipulatives were introduced using questioning to engage students. As the lessons developed, worksheets were used to develop understanding of mathematical concepts. The lessons concluded with questioning to guide student reflection on what had been learnt.

Limitations

There were two major limitations during this research. One of the limitations involved the ability levels of the students. Being an academic, or high achieving class, there were a number of students who did not feel the need to use manipulatives as they understood the concepts without them. This made it

difficult to assess whether the manipulatives worked for the class as a whole. In addition, limited resources were available for this research. For example, there were not enough clocks for each student so some students' choice not to use manipulatives was due to limited access.

Findings and discussion

Engagement

For the purpose of this study, the level of student engagement was measured by a decrease in distractions such as off task behaviour, calling out and leaving workstations. Observation of students during lessons on 'time' and on '3-dimensional objects' revealed that the number of instances of students being off task, getting out of their seats or calling out was reduced when manipulatives were used. Observations of the two lessons recorded in Table 1 reveal that the number of times students were off task, out of their seat or calling out increased from 38 to 62 (63% increase) as the teaching method changed from including to excluding manipulatives.

In terms of behaviour management, this research shows that having students work with physical objects as individuals or in groups not only significantly improves student engagement but also reduces instances of calling out and leaving workstations. It is interesting, however, that the most significant differences were in student engagement during the introduction and development sections of the lesson. During the lessons that included manipulatives there was an average of 14.5 instances of disruptive behaviour per lesson during the introduction and development stages compared to 26 instances per lesson in those that excluded manipulatives. However, there was no real difference in the amount of disruptive behaviour during the reflection part of the lessons (4.5 instances per lesson with manipulatives and five instances per lesson without manipulatives).

A finding that surfaced in this study that was not anticipated but is very interesting is the difference in student engagement when using two dimensional as opposed to three dimensional manipulatives. In an era where students are exposed to technology and two dimensional versions of virtual reality, this study revealed that the cohort of students involved in the research were more engaged with three dimensional physical manipulatives than with interactive whiteboard activities or any other form of virtual manipulatives.

Table 1: Observation record of occurrences of disruptive behaviour during four Mathematics lessons. Two lessons employed the use of manipulatives whilst the other two did not.

	Lesson topic: Time			Lesson topic: 3D objects		
	Off task	Getting out of seat	Calling out	Off task	Getting out of seat	Calling out
Lesson Stages	With manipulatives			With manipulatives		
Introduction using IWB	2	1	4	1	2	5
Development using manipulatives and worksheet	3	2	0	3	3	3
Reflection	2	1	1	2	0	3
Total	**16**			**22**		
Lesson Stages	Without manipulatives			Without manipulatives		
Introduction using questioning	4	3	4	4	2	6
Development using worksheet	6	3	5	7	3	5
Reflection	2	1	3	2	0	2
Total	**31**			**31**		

Understanding and Efficiency

Studying the samples of the students' work was a means of triangulating the results obtained from the observations and surveys. The steps students took to arrive at answers and the accuracy of their work indicated that the use of manipulatives aided their understanding and efficiency. Work samples from a Year 3 student during the time unit showed that accuracy decreased in the lesson that did not have a clock available as a manipulative.

Students were surveyed with regard to how they believed manipulatives aided their understanding. The question was: Maths objects help me understand Maths better. They were asked to answer from options of always, usually, never and sometimes. It can be seen from Figure 1 that 88% of students agreed that their understanding was enhanced with manipulatives either always or usually.

Figure 1: Students' responses to whether manipulatives help them understand better.

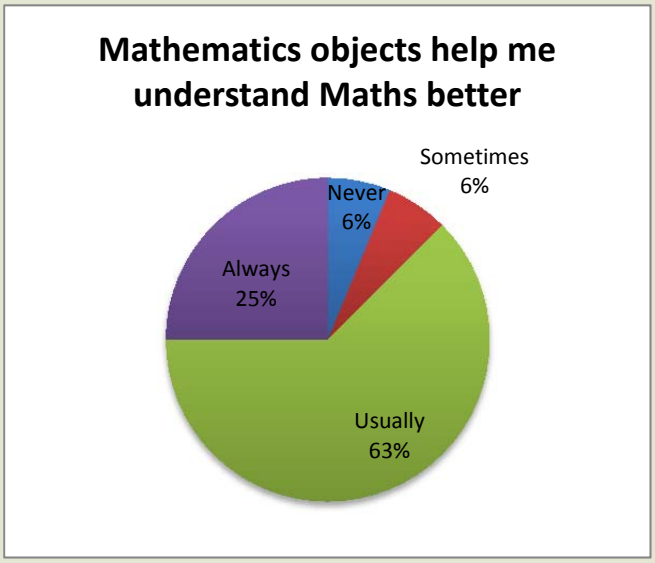
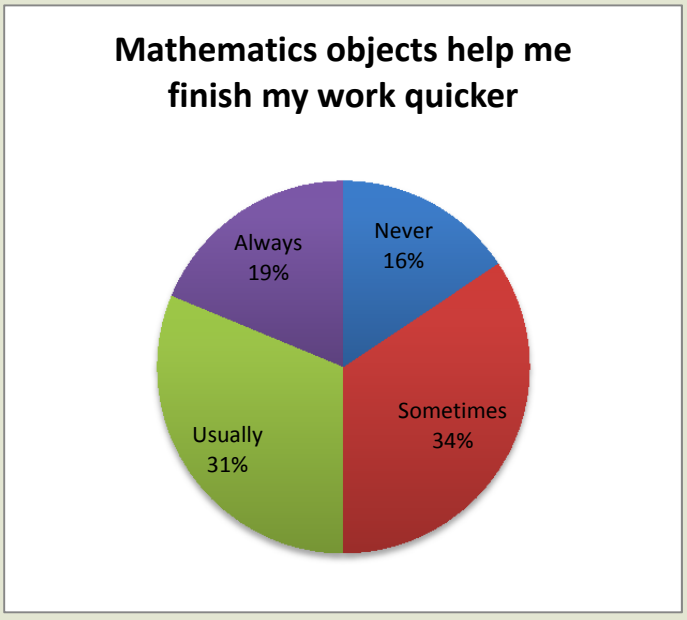


Figure 2: Students' responses to whether manipulatives help them finish their work quicker.

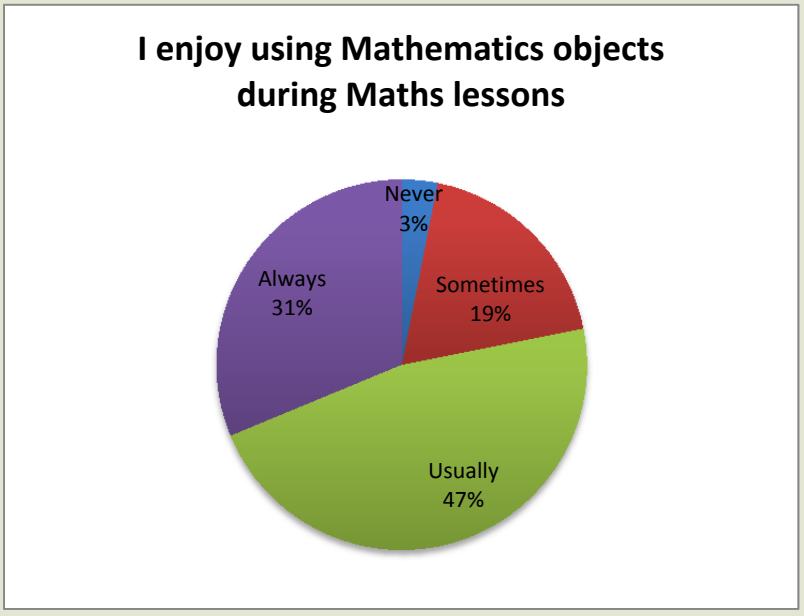


To find out whether students believed manipulatives made them work more efficiently, they were asked whether mathematics objects helped them finish their work quicker. Their responses shown in Figure 2 show that 50% of students indicated that this was the case always or usually but only 16% said this was never the case.

Enjoyment

To find out the level of student enjoyment using manipulatives, students were asked to respond to the statement, I enjoy using mathematics objects during Maths lessons. It is clear from this data, as shown in Figure 3, that there were very few students who answered in the negative (3%). In fact, 78% of students indicated that they always or usually enjoy using objects in Mathematics classes.

Figure 3: Students' responses to whether they enjoy using mathematics manipulatives.



Summary

This research has confirmed most of what the literature has put forward on the place of mathematics manipulatives in the primary classroom. Also indicated by this study is the fact that not only were the students observed to be more engaged in the learning process, but their perception of their classroom environment, or the way they relate to the activities and tone of the classroom, were largely very positive when using manipulatives.

This study found that students generally enjoyed the use of manipulatives within a Mathematics session. Whether students actually needed the manipulatives to grasp the concept depended on the grade level and Mathematics topic being covered. During the time unit it was noted that most students needed the manipulative of the clock to help them solve questions. Similarly, in the length unit, nearly all students needed the manipulative of the ruler to help them solve questions to do with measurement.

Recommendations for practice and future research

This was a small-scale action research that was limited both by sample size and time period in which the data was collected. Thus the results of the study are reported cautiously. This study indicates that manipulatives are a useful tool and that they should be included in Mathematics lessons where suitable.

This research was done in an academic class where the manipulatives were offered as a choice for the students. Further study needs to be done with a range of ages and abilities across the primary sector so that the results of this study can be compared with, for example, a lower grade where students need manipulatives to visualise Mathematics concepts.

Also noted but not recorded in this study was the use of concrete 3D objects. Students needed the hands on materials to solve questions such as: vertices, corners, sides etc. It was interesting to see that students preferred hands on games rather than interactive games on the Interactive whiteboard but from observations it was noted that students grasped Mathematics concepts better when the interactive whiteboard was used as a manipulative at the start of the lesson rather than not using it at all. This finding deserves further study.

Conclusion

It was found that students were more engaged when manipulatives were used rather than when they were not used. In general it seems that the majority of students enjoy the use of manipulatives and manipulatives improve their development of mathematics concepts. The perception of students is that mathematics manipulatives help them be more efficient in their work, better understand their work and receive greater enjoyment from the learning process.

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